Midterm 1, MAT 324, October 25, 2006

Answer each question on the paper provided. Write neatly and give complete answers. Each question is worth 10 points.

1. Define Borel set.
2. Define Lebesgue measurable set.
3. Prove that if A is measureable, then $E = \{|x| : x \in A\}$ is measureable.
4. Prove that that $m^*(tA) = tm^*(A)$ for any set A and real number t.
5. Prove that if A is measurable and $m^*(A \Delta B) = 0$ then B is measurable.
6. Show the collection of intervals $(a, b]$ generates the same σ-field as the intevals (a, b) generate.
7. If f g are measurable, show $h(x) = \max(f(x), g(x))$ is measureable.
8. If every $\{f_n\}$ is measureable show that the set where $\lim_{n\to\infty} f_n(x)$ converges is measureable.
9. If f is a simple function, state the two definitions of $\int_R f dm$ and show they agree.
10. Show that $\text{esssup}(f + g) \leq \text{esssup} f + \text{esssup} g$.