## PROBLEM SET 5

1. Does $\{\sin(nx)\}$ converge in the $L^1$ norm?

2. Give an example of a sequence of functions $\{f_n\}$ which converges to the constant zero function in $L^1$, but so that $f_n(x)$ does not converge to zero at any point of $[0,1]$.

3. If $f_n \to f$ in the $L^1$ norm, show that there is a subsequence $f_{n_k}$ which converges a.e. to $f$.

4. Prove that the set of continuous functions of compact support is dense in $L^1$.

5. For each $1 \leq p \leq \infty$ give an example of a function which is in $L^p$ but not in $L^q$ for any $q \neq p$.

6. If $f \in L^1$ show that $X_f = \{g \in L^1 : |g| \leq |f|\}$ is a compact set in the $L^1$ topology (i.e., a sequence in this set has a subsequence converging to a point of the set).

7. If $f \in L^2$ let $T_n f$ be the $n$th partial sums of the Fourier series. Then $T_n f \to f$ in the $L^2$ norm. It is also true that $T_n f \to f$ a.e., but this is very hard to prove (one of the most famous theorems proven in the 20th century).