PROBLEM SET 3

1. If f is measurable, show that the set of local maxima of f is a measureable set (x is a local maximum of f if there is an interval I centered at x so that $f(x) = \max_{y \in I} f(y)$).

2. Suppose f is continuous on the reals and let $f^(2) = f \circ f$ and $f^{(n)} = f \circ f^{(n-1)} = f \circ f \circ \ldots \circ f$ n times. Let $F(x) = 0$ if $\{x_n\} = \{f^n(x)\}_{n=1}^{\infty}$ is bounded and $F(x) = 1$ if the sequence is unbounded. Show F is measurable.

3. A function is called simple if it only takes on finite number of different values. If g is bounded and measurable, and $\epsilon > 0$ is given, show there is a measurable simple function f so that $\sup_x |g(x) - f(x)| \leq \epsilon$. Is this true if g is not bounded?