SAMPLE MIDTERM 2 MAT 142
Midterm will be Thursday, March 30, 2000

1. Place the letter corresponding to the correct answer in the box next to each question.

(i) $\square$ Which of the following grows fastest as $x \to \infty$/ **(a)** x^2 **(b)** e^x **(c)** e^{x^2} **(d)** x^{100} **(e)** $\ln x$ **(f)** xe^x

(ii) $\square$ $\cos(\arctan(x)) =$**(a)** $1/\sqrt{1+x^2}$ **(b)** $1/\sqrt{1-x^2}$ **(c)** $1+x^2$ **(d)** $\sqrt{1-x^2}$ **(e)** $1-x^2$ **(f)** none of these.

(iii) $\square$ $\int \frac{du}{\sqrt{1-u^2}} =$ **(a)** $\arctan(u)+C$ **(b)** $\arcsin(u)+C$ **(c)** $\arccos(u)+C$ **(d)** $\ln|1-u^2|+C$ **(e)** arcsec(u) + C **(f)** none of these.

(iv) $\square$ $\cosh x =$ **(a)** e^x **(b)** $e^x - e^{-x}$ **(c)** $\frac{1}{2}(e^x + e^x)$ **(d)** $1 - \sinh^2 x$ **(e)** $\frac{1}{2}(e^{-x} - e^x)$ **(f)** none of these.

(v) $\square$ Which of the following is a solution of $y' = y^2$ **(a)** $y = x$ **(b)** $y = 1/x$ **(c)** $y = e^x$ **(d)** $y = -1/x$ **(e)** $y = e^{-x}$ **(f)** none of these.

(vi) $\square$ Use Euler's method with step size $\Delta x = 1$, on the inital value problem $y' = y+x$, $y(0) = 0$ to estimate $y(2)$. **(a)** 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** 4 **(f)** none of these.

(vii) $\square$ Which of the following equations is separable? **(a)** $\frac{dy}{dx} = y + \sin x$ **(b)** $\frac{dy}{dx} = \sin(y+x)$ **(c)** $\frac{dy}{dx} = y \sin x$ **(d)** $\frac{dy}{dx} = \sin(xy)$ **(e)** $\frac{dy}{dx} = x + y$ **(f)** none of these.

(viii) $\square$ Suppose $\frac{x^2+1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$. Then $A =$ **(a)** -2 **(b)** -1 **(c)** 0 **(d)** 1 **(e)** 2 **(f)** none of these.

(ix) $\square$ Which of the following improper integrals diverges? **(a)** $\int_0^\infty e^{-x}dx$ **(b)** $\int_0^\infty (1+x^2)^{-1}dx$ **(c)** $\int_1^\infty \frac{x}{1+x^2}dx$ **(d)** $\int_1^\infty \frac{x}{1+x^3}dx$ **(e)** $\int_0^\infty \frac{\sin x}{1+x^2}dx$ **(f)** none of these.

(x) $\square$ If we use the substitution $x2\tan\theta$, the integral $\int \frac{dx}{\sqrt{4+x^2}}$ becomes **(a)** $\int \sec\theta d\theta$ **(b)** $\int \tan\theta d\theta$ **(c)** $\int \sin^2\theta d\theta$ **(d)** $\int \sec\theta\tan\theta d\theta$ **(e)** $\int \sec^2\theta d\theta$ **(f)** none of these.

2. Find each of the following integrals.

   (i) $\int \frac{xdx}{\sqrt{8x^2+1}}$

   (ii) $\int x^2 \ln x dx$

   (iii) $\int \frac{x}{x^2-3x+2} dx$

   (iv) $\int_{-\infty}^{\infty} \frac{4}{x^2+16}$

   (v) $\int \frac{1}{1+\sin x} dx$ (Hint: multiply and divide by $1 - \sin x$)

3. Solve the differential equation $(1+x)y' + y = \sqrt{x}$.

4. Find a positive number a such that $\int_0^a \frac{dx}{1+x^2} = \int_a^{\infty} \frac{dx}{1+x^2}$.

5. Find the length of the curve $y = \ln x$ from $x = 1$ to $x = e$.