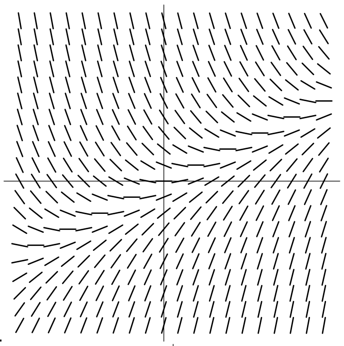
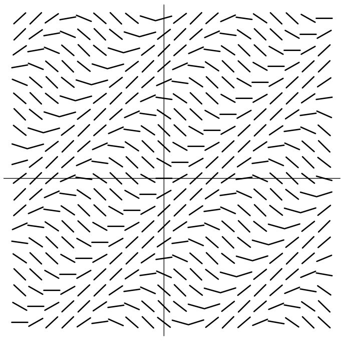
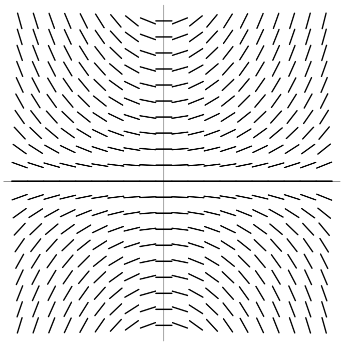
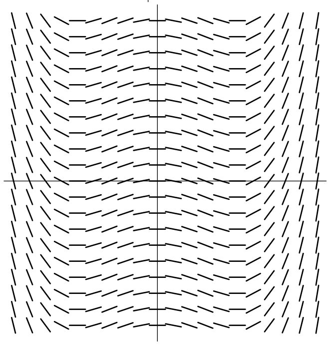
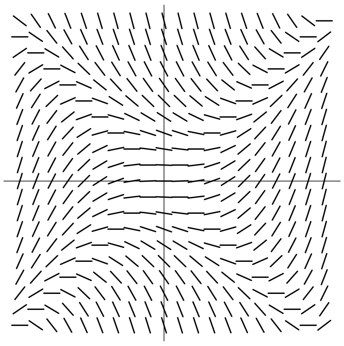
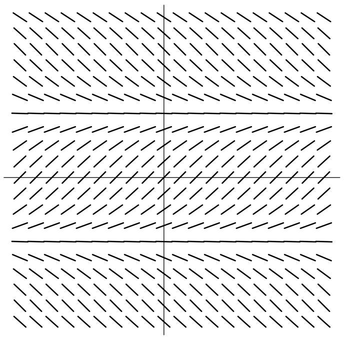
Sample Midterm 2, MAT 142, Fall 2005
First Midterm is Friday, Nov 11, at the usual time and place

1. Put the letter of the slope field in the box of the corresponding equation. All slope fields are graphed on $[-2,2]\times[-2,2]$.

A  B  C 

D  E  F 

i: $\square$ $y' = x^2 - y^2$ **iii:** $\square$ $y' = \cos(2y)$ **v:** $\square$ $y' = x^3 - x$

ii: $\square$ $y' = x - 2y$ **iv:** $\square$ $y' = xy$ **vi:** $\square$ $y' = \sin(3(x-y))$

2. Match each differential equation to the corresponding solution.

i: $\square$ $1/x$ **iii:** $\square$ $\sqrt{1+x}$

ii: $\square$ e^{2x} **iv:** $\square$ $\cos(x)$

A: $y' = -y/x$ **C:** $y'' = 4y$
B: $y' = 1/(2y)$ **D:** $y' = -\sqrt{1-y^2}$

3. Put a 'C' (for converges) or 'D' (for diverges) in the box next to each infinite series and explain why this is correct using tests from the textbook.

(a) $\square$ $\sum_{n=1}^{\infty} \frac{n^2}{n^3+10}$

(b) $\square$ $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^2}$

(c) $\square$ $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

(d) $\square$ $\sum_{n=1}^{\infty} \sin(2^{-n})$

(e) $\square$ $\sum_{n=1}^{\infty} a_n$, where $a_1 = 1$ and $a_{n+1} = \frac{1}{3}(a_n + a_n^2)$ for $n > 1$.

4. Evaluate each of the following infinite series.
   (a) $\sum_{n=1}^{\infty} 5^{-n}$
   (b) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$
   (c) $\sum_{n=1}^{\infty} \ln(\frac{1+2^{-n-1}}{1+2^{-n}})$
   (d) $\sum_{n=0}^{\infty} e^{n}$
   (e) $1 + x + y^2 + x^3 + y^4 + x^5 + y^6 + \dots$
5. Solve each of the following differential equations.
   (a) $2\sqrt{xy}\frac{dy}{dx}, x, y > 0$
   (b) $\frac{dy}{dx} = e^{x-2y}$
   (c) $\frac{dy}{dx} = \sqrt{y}\cos^2(\sqrt{y})$
   (d) $2y' = e^{x/2} + y$
   (e) $(x-1)^3 y' + r(x-1)^2 y = x + 1, x > 1$
6. A tank contains 100 gallons of brine in which 50 lbs or salt are disolved. A brine containing 2 lbs/gal of salt runs into the tank at a rate of 5 gals/min. The solution is kept uniformly mixed and flows outof the tank at 4 gals/min. Write down and solve the intial value problem described by the mixing process.
7. Apply Euler's method with $n = 4$ to the intial value problem $y' = y + 1$, $y(0) = 1$ to estimate $y(1)$. Find the exact value of $y(1)$. Which is larger?
8. Let $a_n = 2^{-n}$ if n is even and $a_n = n/2^n$ if n is odd. Does $\sum_{n=1}^{\infty} a_n$ converge? Explain why.
9. Suppose $\{a_n\}$ is a sequence which converges to 0. Prove there is a subsequence $\{b_n\}$ of $\{a_n\}$ so that $\sum_{n=1}^{\infty} b_n$ converges.