Sample Midterm 1, MAT 142, Fall 2005
First Midterm is Friday, Oct 7, at the usual time and place

(1) Place the letter corresponding to the correct answer in the box next to each question.

(i) $\square$ Which of the following functions is not one to one on the interval $(0,1)$?
(a) $1/x$ **(b)** $\cos x$ **(c)** x^2 **(d)** $\sin 2x$ **(e)** $\ln x$ **(f)** none of these.

(ii) $\square$ Evaluate $\int_0^1 \frac{1}{1+x^2}dx$.
(a) $\pi/4$ **(b)** $\pi/3$ **(c)** $\pi/2$ **(d)** $\frac{3}{4}\pi$ **(e)** π **(f)** none of these.

(iii) $\square$ If we rewrite $4x - x^2$ by completing the square we get
(a) $(x+2)^2-4$ **(b)** $(x-2)^2-4$ **(c)** $(x-2)^2+4$ **(d)** $4-(x-2)^2$ **(e)** $4-(x+2)^2$ **(f)** none of these.

(iv) $\square$ What is $\frac{d}{dx}(\sec^{-1}(x))$?
(a) $|x|/\sqrt{x^2-1}$ **(b)** $1/(|x|\sqrt{x^2+1})$ **(c)** $|x|/\sqrt{x^2+1}$ **(d)** $1/\sqrt{x^2-1}$ **(e)** $1/(|x|\sqrt{x^2-1})$. **(f)** none of these.

(v) $\square$ The error estimate for Simpson's rule with n subdivisions is of the form a constant (depending on f and the interval) times:
(a) n^{-1} **(b)** n^{-2} **(c)** n^{-3} **(d)** n^{-4} **(e)** n^{-5} **(f)** none of these.

(vi) $\square$ The function $\frac{1}{x}\ln x$ has an absolute maximum at $x =$
(a) 0 **(b)** 1 **(c)** $e/2$ **(d)** e **(e)** $2e$ **(f)** none of these.

(vii) $\square$ What is the definition of $\cosh x$?
(a) $2/(e^x - e^{-x})$ **(b)** $\frac{1}{2}(e^x - e^{-x})$ **(c)** $\frac{1}{2}(e^x + e^{-x})$ **(d)** $(e^x - e^{-x})/(e^x + e^x)$ **(e)** $(e^x + e^{-x})/(e^x - e^{-x})$ **(f)** none of these.

(viii) $\square$ Find the inverse function of $f(x) = (1+x)^3$
(a) $(1+x)^{-3}$ **(b)** $(1-x)^3$ **(c)** $x^{1/3}-1$ **(d)** $(x-1)^{1/3}$ **(e)** $(1+x)^{1/3}$ **(f)** none of these.

(ix) $\square$ The function $f(x) = ax + \sin(x)$ is one to one
(a) for all values of a **(b)** for no values of a **(c)** only if $a > 0$ **(d)** only if $a \le -1$ or $a \ge 1$ **(e)** only if $-1 \le a \le 1$ **(f)** none of these.

(x) $\square$ Find the derivative of $f(x) = (\ln x)^3$
(a) $3(\ln x)^2/x$ **(b)** $3(\ln x)^2$ **(c)** $2\ln x$ **(d)** $3/x^2$ **(e)** $(\ln x)/x$ **(f)** none of these.

(2) Find each of the following integrals.
  (i) $\int_2^3 \frac{1}{x} dx$
  (ii) $\int_1^2 \frac{2 \ln x}{x} dx$
  (iii) $\int_{-\pi/2}^{\pi/2} \cos^3(x) dx$
  (iv) $\int \frac{dy}{y^2 - 2y + 5}$
  (v) $\int \sin(x) e^{\cos(x)} dx$
(3) Do each of the following
  (i) Expand by partial fractions: $\frac{x+4}{x^2+5x-6}$
  (ii) Expand by partial fractions: $\frac{1}{(x+1)(x^2+1)}$
  (iii) Explain whether the integral converges or diverges: $\int_0^1 \ln(x) dx$
  (iv) Explain whether the integral converges or diverges: $\int_0^\infty \frac{x^2}{1+x^2+x^3} dx$
  (v) Explain whether the integral converges or diverges: $\int_0^1 x^{-\alpha} dx, 0 < \alpha < 1$
(4) How many divisions are required for the trapezoid rule to evaluate $\int_0^1 e^x dx$ with error less than .0001?
(5) Using the definitions of the hyperbolic functions, prove the identity

$$\cosh^2 x = \frac{1}{2}(\cosh 2x + 1).$$

(6) Show that for $|x| < 1$, $\frac{d}{dx} \tanh^{-1}(x) = \frac{1}{1-x^2}$
(7) What values of $p \in (0, \infty)$ have the property that the volume of the solid obtained by rotating the region $\{(x, y) : x > 1, y < x^{-p}\}$ around the x-axis is finite?