SAMPLE MIDTERM 2 MAT 142
Midterm will be Thursday, Nov 15, 2001

1. Place the letter corresponding to the correct answer in the box next to each question.

(i) $\square$ If we divide $3x+2$ into $3x^2-7x$, the remainder term is **(a)** 0 **(b)** -1 **(c)** 1 **(d)** 6 **(e)** 2 **(f)** none of these.

(ii) $\square$ What is the limit of the sequence given by $a_n = n^{1/n}$? **(a)** 0 **(b)** 1/2 **(c)** 1 **(d)** e **(e)** e^2 **(f)** none of these.

(iii) $\square$ Suppose $\frac{x-1}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$. Then $C =$ **(a)** -2 **(b)** -1 **(c)** 0 **(d)** 1 **(e)** 2 **(f)** none of these.

(iv) $\square$ Which of the following improper integrals converges? **(a)** $\int_0^1 \frac{1}{x}dx$ **(b)** $\int_0^1 x^{-2}dx$ **(c)** $\int_0^1 x^{-1/2}dx$ **(d)** $\int_1^\infty \frac{1}{x}dx$ **(e)** $\int_1^\infty x^{-1/2}dx$ **(f)** none of these.

(v) $\square$ Suppose $\frac{x^2+1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$. Then $A =$ **(a)** -2 **(b)** -1 **(c)** 0 **(d)** 1 **(e)** 2 **(f)** none of these.

(vi) $\square$ Which of the following improper integrals diverges? **(a)** $\int_0^\infty e^{-x}dx$ **(b)** $\int_0^\infty (1+x^2)^{-1}dx$ **(c)** $\int_1^\infty \frac{x}{1+x^2}dx$ **(d)** $\int_1^\infty \frac{x}{1+x^3}dx$ **(e)** $\int_0^\infty \frac{\sin x}{1+x^2}dx$ **(f)** none of these.

(vii) $\square$ If we use the substitution $x = \tan\theta$, the integral $\int \frac{dx}{\sqrt{4+x^2}}$ becomes **(a)** $\int \sec\theta d\theta$ **(b)** $\int \tan\theta d\theta$ **(c)** $\int \sin^2\theta d\theta$ **(d)** $\int \sec\theta\tan\theta d\theta$ **(e)** $\int \sec^2\theta d\theta$ **(f)** none of these.

(viii) $\square$ Suppose f is positive and decreasing and $\int_0^\infty f(x)dx$ converges. Then which of the following must be true?
(a) $\int_0^\infty f^2(x)dx$ must converge **(b)** $\int_0^\infty f^2(x)dx$ must diverge **(c)** $\lim_{x\to\infty} f(x) > 0$ **(d)** $\lim_{x\to\infty} f(x)/x = 1$ **(e)** $\int_0^\infty xf(x)dx$ must diverge **(f)** none of these.

(ix) $\square$ What is the limit of the sequence $\sqrt{n}/\sqrt{n+2}$? **(a)** 0 **(b)** 1/2 **(c)** $1/\sqrt{2}$ **(d)** 1 **(e)** 2 **(f)** none of these.

(x) $\square$ Evaluate the series $1 - 2x + 4x^2 - 8x^3 + 16x^4 - 32x^5 + \dots$ where it converges. **(a)** $\frac{1}{1-x}$ **(b)** $\frac{1}{1-2x}$ **(c)** $\frac{1}{1+2x}$ **(d)** $\frac{1}{1+x^2}$ **(e)** $\frac{2}{1-x}$ **(f)** none of these.

2. Find each of the following integrals.

   (i) $\int x^2 \sin x dx$

   (ii) $\int \frac{x}{(x-1)(x-2)} dx$

   (iii) $\int \frac{dx}{x^2\sqrt{x^2+1}}$

   (iv) $\int \frac{1-x}{\sqrt{1-x^2}} dx$

   (v) $\int \frac{1}{1+\sin x} dx$ (Hint: multiply and divide by $1 - \sin x$)

3. Determine whether each of the following infinite series converges or diverges (you do not need to find the limit if it converges) and explain why.

   (i) $\sum_{n=1}^{\infty} \frac{n^2}{3+n^3}$

   (ii) $\sum_{n=1}^{\infty} n^3 3^{-n}$

   (iii) $\sum_{n=1}^{\infty} n^{-n}$

   (iv) $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n})$

4. Evaluate each of the following limits.

   (i) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

   (ii) $\lim_{x\to 0^+} x \ln x$

   (iii) $\lim_{x\to 1} (x^2 - 2x + 1)^{x-1}$

   (iv) $\lim_{x\to\infty} \frac{3x-5}{2x^2-x+2}$

   (v) $\lim_{x\to\infty} (1 + \frac{1}{x})^x$.

5. For what values of p does the series $\sum_{n=1}^{\infty} \frac{n^p}{\sqrt{n^3+1}}$ converge? Justify your answer.

6. Find the sum of the infinite series $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$. Justify your answer.

7. Prove that if $\{a_n\}$ is a bounded sequence then it has a subsequence which is monotone.