Sample Midterm 1, MAT 142, Fall 2001
First Midterm is Thursday, Oct 11, at the usual time and place

1. Place the letter corresponding to the correct answer in the box next to each question. Each correct answer is worth 2 points.

   (i) ☐ Which of the following functions is not one to one on the interval $(0, 1)$?
   (a) $1/x$ **(b)** $\cos x$ **(c)** x^2 **(d)** $\sin 2x$ **(e)** $\ln x$ **(f)** none of these.

   (ii) ☐ Evaluate $\int_0^1 \frac{1}{1+x^2} dx$.
   (a) $\pi/4$ **(b)** $\pi/3$ **(c)** $\pi/2$ **(d)** $\frac{3}{4}\pi$ **(e)** π **(f)** none of these.

   (iii) ☐ If we rewrite $4x - x^2$ by completing the square we get
   (a) $(x + 2)^2 - 4$ **(b)** $(x - 2)^2 - 4$ **(c)** $(x - 2)^2 + 4$ **(d)** $4 - (x - 2)^2$ **(e)** $4 - (x + 2)^2$ **(f)** none of these.

   (iv) ☐ Which of the following is a solution of $y' = \frac{y}{x} + 1$
   (a) $y = x \ln x$ **(b)** $y = xe^x$ **(c)** $y = \ln x$ **(d)** $y = e^{x+1}$ **(e)** $y = x^2 + 1$ **(f)** none of these.

   (v) ☐ The function $y = e^{x^2}$ is a solution of which of the following differential equations?
   (a) $y' = x^2 y$ **(b)** $y' = \frac{1}{y} \ln y$ **(c)** $y' = y \ln y$ **(d)** $y' = xy$ **(e)** $y' = 2xy$ **(f)** none of these.

   (vi) ☐ Which of the following differential equations is not separable?
   (a) $y' = x^2 y$ **(b)** $y' = 1 + x + y + xy$ **(c)** $y' = e^{x+y}$ **(d)** $y' = x + y$ **(e)** $y' = y^2 + 1$ **(f)** none of these.

   (vii) ☐ What is the definition of $\cosh x$?
   (a) $2/(e^x - e^{-x})$ **(b)** $\frac{1}{2}(e^x - e^{-x})$ **(c)** $\frac{1}{2}(e^x + e^{-x})$ **(d)** $(e^x - e^{-x})/(e^x + e^x)$ **(e)** $(e^x + e^{-x})/(e^x - e^{-x})$ **(f)** none of these.

   (viii) ☐ Find the inverse function of $f(x) = (1 + x)^3$
   (a) $(1 + x)^{-3}$ **(b)** $(1 - x)^3$ **(c)** $x^{1/3} - 1$ **(d)** $(x - 1)^{1/3}$ **(e)** $(1 + x)^{1/3}$ **(f)** none of these.

   (ix) ☐ The function $f(x) = ax + \sin(x)$ is one to one
   (a) for all values of a **(b)** for no values of a **(c)** only if $a > 0$ **(d)** only if $a \leq -1$ or $a \geq 1$ **(e)** only if $-1 \leq a \leq 1$ **(f)** none of these.

   (x) ☐ Find the derivative of $f(x) = (\ln x)^3$
   (a) $3(\ln x)^2/x$ **(b)** $3(\ln x)^2$ **(c)** $2 \ln x$ **(d)** $3/x^2$ **(e)** $(\ln x)/x$ **(f)** none of these.

2. Find each of the following integrals.

   (i) $\int_2^3 \frac{1}{x} dx$

   (ii) $\int_1^2 \frac{2\ln x}{x} dx$

   (iii) $\int \frac{1}{9+3x^2} dx$

   (iv) $\int \frac{dy}{y^2-2y+5}$

   (v) $\int \sin(x) e^{\cos(x)} dx$

3. Solve each of the following differential equations or initial value problems.

   (i) $\frac{dy}{dx} = x^2\sqrt{y}$

   (ii) $\frac{dy}{dx} = \sqrt{y}\cos^2(\sqrt{y})$

   (iii) $x\frac{dy}{dx} + y = e^x, x > 0$

   (iv) $e^x \frac{dy}{dx} + 2e^x y = 1$

   (v) $\frac{dy}{dx} + 2y = 3, y(0) = 1$

4. Newton's law of cooling says $T - T_S = (T_0 - T_S)e^{-kt}$, where T_0 is the initial temperature and T_S is the surrounding temperature. Use this to solve the following problem. Suppose that a cup of soup cooled from 90^o C to 60^o C after 10 minutes in a room whose temerature is 20^o C. How much longer would it take the soup to cool to 35^o C?

5. A 200 gallon tank is half full of distilled water. At time $t = 0$, a solution containing 0.5 lb/gal of concetrate enters the tank at a rate of 5 gal/min and the well stirred mixture is withdrawn at a rate of 3 gal/min. At the time the tank is full, how many pounds of concentrate will it contain?

6. Using the definitions of the hyperbolic functions, prove the identity $\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$.

7. Show that for $|x| < 1$, $\frac{d}{dx}\tanh^{-1}(x) = \frac{1}{1-x^2}$

8. Show that every solution of the differential equation $y' = -y^3$ tends to zero as $t \to +\infty$.