SAMPLE MIDTERM 2 MAT 141

The second midterm will be on Friday, November 19 at the usual class time (12:40pm). Section 1 will take the exam in room 201 of Heavy Enginnering (same as last time) and Section 2 will take in our usual lecture room, room 152 of Light Enginnering.

1. Place the letter corresponding to the correct answer in the box next to each question.

(i) ☐ What is the slope of the curve given by $x^3 + y^3 - 9xy = 0$ at the point $(x, y) = (2, 4)$? **(a)** 1 **(b)** $\frac{24}{30}$ **(c)** $\frac{3}{4}$ **(d)** $\frac{9}{18}$ **(e)** $\frac{6}{5}$ **(f)** none of these.

(ii) ☐ Suppose $f(x) = |x^2 - 2x|$. The set of critical points of f is
(a) $\{0\}$ **(b)** $\{1\}$ **(c)** $\{0, 1, 2\}$ **(d)** $\{2\}$ **(e)** $\{0, 2\}$ **(f)** none of these.

(iii) ☐ Suppose $g'(x) = \sin^{1999}(x)$. The absolute maximum of g on $[0, 2\pi]$ occurs at
(a) 0 **(b)** $\pi/4$ **(c)** $\pi/2$ **(d)** π **(e)** 2π **(f)** none of these.

(iv) ☐ What is $\lim_{x\to+\infty} \frac{x^2+2}{x^3+x^2+x}$?
(a) 0 **(b)** 1 **(c)** $\frac{2}{3}$ **(d)** $\frac{1}{2}$ **(e)** $+\infty$ **(f)** none of these.

(v) ☐ Find the linearization of $f(x) = x^3 - x$ at $x = 1$.
(a) $L(x) = 2x$ **(b)** $L(x) = 2(x + 1)$ **(c)** $L(x) = -2(x - 1) + 1$ **(d)** $L(x) = 2x + 1$ **(e)** $L(x) = 2(x - 1)$ **(f)** none of these.

(vi) ☐ Use differentials to estimate the change in the surface area of a cube $S = 6x^2$ when the edge length goes from x_0 to $x_0 + dx$
(a) $6dx$ **(b)** $6x_0dx$ **(c)** $12x_0dx$ **(d)** $12dx$ **(e)** $18x_0dx$ **(f)** none of these.

(vii) ☐ The formula for finding sucessive approximations in Newton's method is
(a) $x_{n+1} = x_n + f(x_n)/f'(x_n)$ **(b)** $x_{n+1} = x_n - f(x_n)/f'(x_n)$ **(c)** $x_{n+1} = x_n + f'(x_n)/f(x_n)$ **(d)** $x_{n+1} = x_n - f'(x_n)/f(x_n)$ **(e)** $x_{n+1} = x_n - f(x_n)f'(x_n)$ **(f)** none of these.

(viii) ☐ The solution of the inital value problem $\frac{dy}{dx} = x + 1$, $y(2) = 3$ is
(a) $y = x + 1$ **(b)** $y = x^2 - x$ **(c)** $y = \frac{1}{2}x^2 + 2$ **(d)** $y = x^2 + x + 1$ **(e)** $y = \frac{1}{2}x^2 + x$ **(f)** none of these.

(ix) ☐ Suppose $f'(x) = x^2 \sin^{10}(x)$. Then on the interval $[0, \frac{1}{2}\pi]$ the function f is
(a) increasing and concave down **(b)** increasing and concave up **(c)** decreasing and concave down **(d)** decreasing and concave up **(e)** constant **(f)** none of these.

(x) ☐ The function $f(x) = x^3 - 3x^2 + 1$ has a point of inflection at $x =$?
(a) -2 **(b)** -1 **(c)** 0 **(d)** 1 **(e)** 2 **(f)** none of these.

2. Find each of the following indefinite integrals

   (i) $\int x^3 - x^2 + 2dx$,

   (ii) $\int \sin(3x)dx$,

   (iii) $\int \cos(3x+2)dx$,

   (iv) $\int \sin^4(t)\cos(t)dt$,

   (v) $\int t(t^2+1)^{1/2}dt$,

3. State the mean value theorem.

4. **(5 pts)** Suppose the second hand on a clock has length 20 cm. At what rate is the distance between the tip of second hand and the 12 o'clock mark changing when the second hand points to 3 o'clock?

5. Suppose it takes 2 hours to replace the drill bit while drilling for oil. A new drill bit digs quickly at first, but slows down with time. Suppose that in t hours it can drill though $f(t)$ feet of rock.

   (i) Suppose the drill bit is used for T hours before being replaced. What is the average speed of drilling (including the 2 hours to install the bit)?

   (ii) Show that to maximize this average speed the bit should should be replaced after T hours of use where T satisfies $f'(T) = f(T)/(T+2)$.

   (iii) If $f(t) = 100t/(t+5)$ find this time T.