SAMPLE FINAL MAT 141, Fall 2000

The final will be 11:00am-1:30pm on Friday, December 22. section 1 will meet in Physics P-118 and section 2 in Physics P-116.

1. Place the letter corresponding to the correct answer in the box next to each question.

(i) ☐ Suppose $g(t) = \sin(t)$. Then the 100th derivative of g is
(a) $\sin(t)$ **(b)** $-\sin(t)$ **(c)** $\cos(t)$ **(d)** $-\cos(t)$ **(e)** $100\sin^{99}(t)$ **(f)** none of these.

(ii) ☐ What is the maximum value of $f(x) = x^2 - x^4$?
(a) $y = 1/16$ **(b)** $y = 1/8$ **(c)** $y = 3/16$ **(d)** $y = 1/4$ **(e)** $y = 1/2$ **(f)** none of these.

(iii) ☐ The error formula for the left hand sum with n subintervals (for monotonic functions) is $E \leq (b-a)|f(b) - f(a)|/n$. Using this formula, what is the minimum n needed to compute $\int_0^3 \sqrt{x^3 + 3x}dx$ with error $\leq .01$?
(a) 36 **(b)** 108 **(c)** 900 **(d)** 1,800 **(e)** 18,000 **(f)** 108,000

(iv) ☐ Solve the initial value problem $y = (x^2+1)/x^2$ and $y(1) = -1$
(a) $y = x - \frac{1}{x}$ **(b)** $y = x + \frac{1}{x} - 3$ **(c)** $y = x - \frac{1}{x} - 1$ **(d)** $y = x - x^{-2} - 1$ **(e)** $y = x^2 - 2$ **(f)** none of these.

(v) ☐ Suppose $G(x) = \int_0^x \sin^2(t)dt$. Then $G''(x) =$
(a) $\sin^2(x)$ **(b)** $2\sin(x)\cos(x)$ **(c)** $-2\sin(x)\cos(x)$ **(d)** $\cos^2(x)$ **(e)** $2\sin(t)$ **(f)** none of these.

(vi) ☐ The area of the region bounded above by $y = \sqrt{x}$ and below by the x-axis and $y = x - 2$ is
(a) $\int_0^4 \sqrt{x} - (x-2)dx$ **(b)** $\int_0^2 \sqrt{x}dx + \int_2^4 \sqrt{x} - (x-2)dx$ **(c)** $\int_0^2 \sqrt{x} - xdx + 2$ **(d)** $\int_0^2 \sqrt{x}dx + \int_2^4 (x-2)dx$ **(e)** $\int_0^4 \sqrt{x}dx$ **(f)** none of these.

(vii) ☐ Suppose $\int_1^3 f(t)dt = 2$. Then $\int_1^3 (2f(t) - 1)dt =$
(a) 0 **(b)** 1 **(c)** 2 **(d)** 3 **(e)** 4 **(f)** none of these.

(viii) ☐ $E = \frac{1}{2}(b-a)^3 \frac{1}{n}^2 M$ is the error term for
(a) left hand sums **(b)** right hand sums **(c)** trapezoid rule **(d)** Simpson's rule **(e)** Calvaleri's rule **(f)** none of these.

(ix) ☐ The area of the region between the graphs of x^2 and x^3 with $0 \leq x \leq 1$ is
(a) 1 **(b)** 1/2 **(c)** 1/3 **(d)** 1/4 **(e)** 1/12 **(f)** none of these.

(x) ☐ The graph $y = f(x)$ is rotated around the x-axis to give a region in 3 space. The volume of the region between $x = 0$ and $x = 1$ is given by the integral
(a) $\pi \int_0^1 f(x)dx$ **(b)** $\int_1^0 (f(x)+\pi)^2 dx$ **(c)** $\int_0^\pi f(x)dx$ **(d)** $\pi \int_0^1 f^2(x)dx$ **(e)** $\int_0^1 (f(x)-\pi)^2 dx$ **(f)** none of these.

2. Find each of the following definite integrals

    (i) $\int_0^1 (x^3 - x^2 + 2)dx$

    (ii) $\int_0^{\pi/4} \sec^2(x)dx$

    (iii) $\int_1^2 (4x + 2)^9 dx$

    (iv) $\int_0^{2\pi} \sin^4(t)\cos(t)dt$

    (v) $\int_{-\pi}^{\pi} \sin(t)\sqrt{1 + \sin^2(t)}dt$

3. Evaluate the following limit: $\lim_{a\to\infty} \int_1^a \frac{1}{x^{1/2}}dx$.

4. The graph of $y = \sqrt{x}$ is rotated around the x-axis to give a region in 3-space. Find the volume of this region between $x = 0$ and $x = 4$.

5. A light shines from the top of a pole 50 ft high. A ball is dropped from the same height from a point 30 ft away. How fast is the shadow of the ball moving along the ground 1/2 second later? (assume the ball falls a distance $16t^2$ ft in t seconds).

6. Find the normals to the curve $xy + 2x + y = 0$ which are parallel to the line $2x + y = 0$.

7. An 1125-ft^3 open-top rectangular tank with square base x ft on a side and y feet deep is to be built with its top flush with the ground to catch runoff water. The costs associated with the tank involve not only the material but also an excavation charge proportional to the product xy. If the cost is $c = 5(x^2 + 4xy) + 10xy$, what values of x and y will minimize the cost?

8. The height of a body moving vertically is given by $s = -\frac{1}{2}gt^2 + v_0t + s_0$ with s in meters and t in seconds. Find the body's maximum height in terms of g, v_0 and s_0. 1

9. Use Simpson's rule with $n = 4$ to approximate $\int_{-\pi/2}^{\pi/2} \cos^2(x)dx$.

10. What is the arclength of the graph of $f(x) = 1 + x + x^3$ between $x = 1$ and $x = 2$? Express the answer as an integral.

11. Suppose the positive part of the graph of $g(x) = 1 - x^2$ is rotated around the x-axis. What is the surface area of the region obtained? Express the answer as an integral.