SAMPLE MIDTERM 1, MAT 141
First midterm is Fri. Oct 13, in regular room and time

1. Place the letter corresponding to the correct answer in the box next to each question.

(i) $\square$ Suppose $0 < a < b < c$. Then which of the following must be true?
(a) $a + b < c$ **(b)** $a^2 + b^2 = c^2$ **(c)** $a^2 + b^2 < c^2$ **(d)** $a(b + c) < c(a + b)$ **(e)** $a - b < a - c$ **(f)** none of these.

(ii) $\square$ What is the equation of the line passing through the points $(-2, 4)$ and $(1, 1)$?
(a) $y - 1 = -(x - 1)$ **(b)** $y + 1 = -(x - 1)$ **(c)** $y - 4 = -(x - 2)$ **(d)** $y + 4 = (x + 2)$ **(e)** $y = -x - 2$ **(f)** none of these.

(iii) $\square$ Suppose that for all $B > 0$ there is a $C > 0$ so that $x > C$ implies $f(x) > B$. Then
(a) $\lim_{x\to 0} f(x) = +\infty$ **(b)** $\lim_{x\to+\infty} f(x) = 0$ **(c)** $\lim_{x\to+\infty} f(x) = 1$ **(d)** $\lim_{x\to+\infty} f(x) = +\infty$ **(e)** $\lim_{x\to 0} f(x) = 0$. **(f)** none of these.

(iv) $\square$ Consider a right triangle with an angle θ, opposite side x and adjacent side 1. What is $\cos(\theta)$?
(a) $1/\sqrt{1 + x^2}$ **(b)** $1/\sqrt{1 - x^2}$ **(c)** $\sqrt{1 + x^2}$ **(d)** $x/\sqrt{1 + x^2}$ **(e)** $x/\sqrt{1 - x^2}$ **(f)** none of these.

(v) $\square$ The derivative of $xh(x^2)$ is
(a) $1+2xh'(x^2)$ **(b)** $h'(x^2)2x$ **(c)** $2x+xh'(x^2)$ **(d)** $xh(x^2)+x^2h'(x)$ **(e)** $h(x^2)+2x^2h'(x)$ **(f)** none of these.

(vi) $\square$ The derivative of $f(x) = x^2 + x^3$ at $x = 2$ is
(a) 12 **(b)** 13 **(c)** 14 **(d)** 15 **(e)** 16 **(f)** none of these.

(vii) $\square$ The natural domain of $f(x) = \frac{\sqrt{x+5}}{x}$ is
(a) all real numbers **(b)** $x > 0$ **(c)** $x < -5$ **(d)** $-5 \le x < 0$ or $0 < x$ **(e)** $x \le 0$ or $x > 5$ **(f)** none of these.

(viii) $\square$ Suppose $f(1) = 3.4$ and $f(1.1) = 3.6$. Then the best estimate for $f'(1)$ is
(a) 3.5 **(b)** 3.4 **(c)** 2.0 **(d)** 20 **(e)** .2 **(f)** .002

(ix) $\square$ A ball dropped from rest takes 3 seconds to hit the ground. From what height was it droped (in feet)?
(a) 48 **(b)** 90 **(c)** 144 **(d)** 256 **(e)** 288 **(f)** none of these

(x) $\square$ What is the limit of $\frac{x^2+\cos x}{2x^2+x+\sin x}$ as $x \to \infty$?
(a) 0 **(b)** $\frac{1}{2}$ **(c)** 1 **(d)** 2 **(e)** ∞ **(f)** the limit fails to exist

2. Evaluate each of the following limits or explain why it does not exist.

   (i) $\lim_{x\to 1} \frac{x^2-1}{x+1}$

   (ii) $\lim_{x\to 0} \frac{(2-x)^2-4}{x}$

   (iii) $\lim_{s\to\infty} \frac{s}{s+1}$

   (iv) $\lim_{h\to 0} \frac{|2-h|}{h}$

   (v) $\lim_{t\to 0} t\sin(\frac{1}{t})$

3. For each of the following functions, find the derivative function.

   (i) $x^{10} + x^{1/2}$

   (ii) $\tan(x)$

   (iii) $x^2 \sin(x)$

   (iv) $\cos(x^2)$

   (v) $(\cos(x) + \sin(x))^3$

4. What are the following limits (you do not need to justify your answer),

$$\lim_{x\to 0} \frac{\sin x}{x} = \boxed{} \qquad \lim_{x\to 0} \frac{\cos x - 1}{x} = \boxed{}$$

   Using these, the quotient definition of derivative and addition law for cosines,

$$\cos(x+h) = \cos(x)\cos(h) - \sin(x)\sin(h),$$

   prove that $\frac{d}{dx}\cos x = -\sin x$.

5. State and prove the product rule for derivatives.

6. Suppose f satisfies the following two conditions for all real values of x and y.

   (i) $f(x+y) = f(x)f(y)$

   (ii) $f(x) = 1 + xg(x)$ where $\lim_{x\to 0} g(x) = 1$.

   Show that f is differentiable at every point and that $f'(x) = f(x)$.