SAMPLE FINAL MAT 123
Spring 2003

room	sections
Gym	R1-R13
Dance studio	ELC 3, ELC 4

FINAL EXAM IS THURSDAY, MAY 14, 11:00am-1:30pm. ROOMS TO BE ANOUNCED IN CLASS. THIS SAMPLE IS SIMILAR TO, BUT SHORTER THAN, THE ACTUAL FINAL.

1. Place the letter corresponding to the correct answer in the box next to each question.

(i) ☐ Simplify $\log_2(4x^2 2^x)$ **(a)** $\ln 2 + 2\ln x + x$ **(b)** $\log_2 2 + 2\log_2 x + x$ **(c)** $\ln 4 + 2\ln x + x\ln 2$ **(d)** $2 + 2\ln x + x$ **(e)** $2 + 2\log_2 x + x$ **(f)** none of these.

(ii) ☐ The initial size of a bacteria colony is 1000. After 1 hour the bacteria count is 8000. Assuming exponential growth, the time it takes the colony to double in size is approximately **(a)** 5 minutes **(b)** 10 minutes **(c)** 15 minutes **(d)** 20 minutes **(e)** 30 minutes **(f)** none of these.

(iii) ☐ Suppose $\sin(t) > 0$. Then at t the function cos must be **(a)** positive **(b)** negative **(c)** increasing **(d)** decreasing **(e)** zero **(f)** none of these.

(iv) ☐ The Richter scale measures the magnitude of an earthquake as $M = \log_{10}(I/S)$ where I is the measured intensity and S is the intensity of a "standard" earthquake. The famous 1906 San Francisco earthquake measured 8.3 on this scale. How much more intense is this than a quake which measures 5.3 on the Richter scale? **(a)** 10 times **(b)** 50 times **(c)** 100 times **(d)** 500 times **(e)** 1000 times **(f)** none of these.

(v) ☐ What is the domain of $\frac{1}{x^2-x}$? **(a)** all x **(b)** all x except 0 **(c)** all x except 0 and 1 **(d)** all $x > 0$ **(e)** $0 \le x \le 1$ **(f)** none of these.

(vi) ☐ Express the following function as an explicit formula: take a number and add 1 to it; then square the result and multiply by 4. **(a)** $f(x) = 4(x+1)^2$ **(b)** $f(x) = (4x+1)^2$ **(c)** $f(x) = 4x^2 + 1$ **(d)** $f(x) = (4x)^2 + 1$ **(e)** $f(x) = 4(x^2+1)$ **(f)** none of these.

(vii) ☐ The period of a pendulum (the time for one complete swing) varies directly with the sqare root of the length of the pendulum. If a pendulum of length L has a period of T seconds, what is the period of a pendulum of length $2L$? **(a)** $2T$ **(b)** $\sqrt{2}T$ **(c)** $T/\sqrt{2}$ **(d)** T^2 **(e)** $\sqrt{T}$ **(f)** none of these.

(viii) ☐ The minimum or maximum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at $x =$ **(a)** $-b/2a$ **(b)** $b/2a$ **(c)** $-b/a$ **(d)** b/a **(e)** a/b **(f)** none of these.

(ix) ☐ A rectanglar box with a volume of 60 ft^3 has a square base. Find a function which models the surface area of the box as a function of x, the length of one side of the base. **(a)** $2x^2 + 4\frac{60}{x}$ **(b)** $x^2 + 240x$ **(c)** $2x^2 + 2\frac{60}{x^2}$ **(d)** $x^2 + \frac{60}{x}$ **(e)** $2x^2 + \frac{4}{x^2}$. **(f)** none of these.

(x) Simplify $\log_{10} 1000$. **(a)** 1 **(b)** 2 **(c)** 1/2 **(d)** 3 **(e)** 4 **(f)** none of these.

(xi) Given that $\ln(12) = 2.48$ and $\ln(3) = 1.10$ what is $\log_3 12$? **(a)** 1.51 **(b)** 2.26 **(c)** 2.57 **(d)** 3.01 **(e)** 3.24 **(f)** none of these.

(xii) If $\sin(x) = 1/\sqrt{2}$ then $\tan(x) =$? **(a)** 0 **(b)** 1/2 **(c)** $1/\sqrt{2}$ **(d)** 1 **(e)** either -1 or 1 **(f)** none of these.

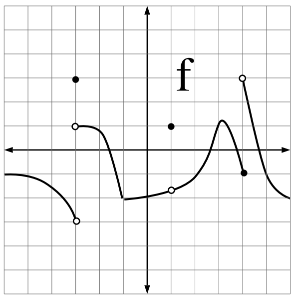
(xiii) Evaluate $\sec(\pi/3)$. **(a)** 0 **(b)** 1/2 **(c)** $\sqrt{3}/2$ **(d)** 1 **(e)** 2 **(f)** none of these.

(xiv) Which of the following identities is **not** true? **(a)** $\csc(x) = 1/\sin(x)$ **(b)** $\sin(\frac{\pi}{2} - x) = \cos(x)$ **(c)** $\cos(x) = \cos(-x)$ **(d)** $\tan(x) = \sin(x)/\cos(x)$ **(e)** $\sec^2(x) + 1 = \tan^2(x)$ **(f)** none of these.

(xv) Suppose f is given by the following table. Estimate the derivative of f at $x = 1$ **(a)** 1 **(b)** 2 **(c)** 3 **(d)** 4 **(e)** 6 **(f)** none of these.

x	0	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
f(x)	1	1.1	1.3	1.7	2.4	2.8	3.2	4.2	4.4	4.6	4.8

2. Answer each of the questions about the function f graphed below. Each box has unit size. In each question, list **ALL** the points a in $[-6, 6]$ so that

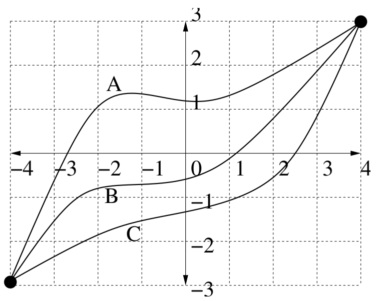


(i) $\lim_{x\to a} f(x)$ exists but does not equal $f(a)$.

(ii) f has a jump at a but $f(a)$ equals the left hand limit at a.

(iii) What is $\lim_{x\to -3^+} f(x)$?

3. The three graphs in the following figure represent the positions of three particles moving along a line between times $t = -4$ and $t = 4$. Answer each of the following questions based on these graphs

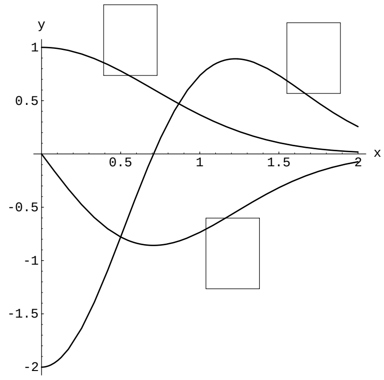


(i) Which particle is moving fastest at time $t = -4$?

(ii) How far apart are A and C at time $t = -1$?

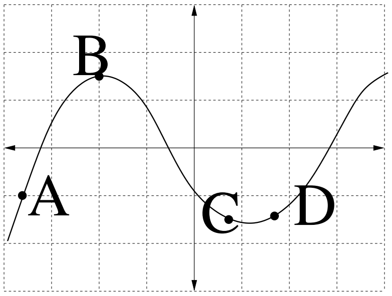
(iii) Which particle has a negative velocity at some time?

4. In the following figure, three functions f, g, h are graphed such that $f = g'$ and $g = h'$. Label each graph correctly.

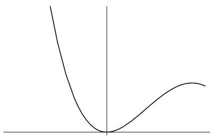


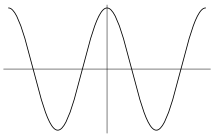
5. Match the letter of each point on the graph with the value which best approximates the slope at that point. Four of the boxes should be left blank.

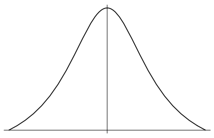
☐ 1	☐ 0
☐ 3	☐ 2
☐ 1/2	☐ -1
☐ -3	☐ $-1/2$

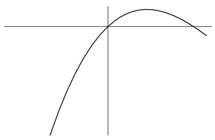


6. For the function graphed on the left, find the corresponding derivative function among any of the choices on the right.

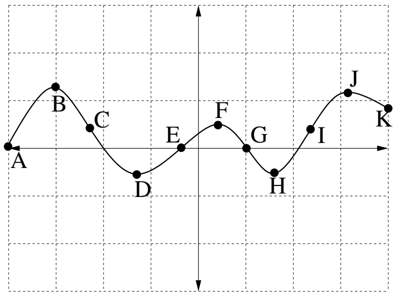
☐ 

A 

B 

C 

7. For each part, list all the labeled points which fit the description for the function graphed below.



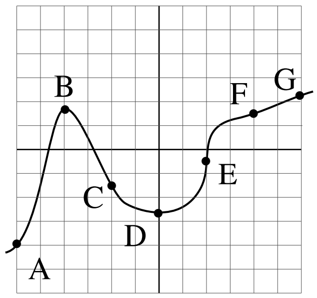
   (i) Where does f take its maximum value?

   (ii) Where is $f'(x) = 0$?

   (iii) Where is $f'(x) > 0$?

   (iv) Where is $f(x) = 0$?

8. Answer each question.



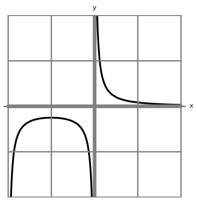
   (i) What is the average rate of change between A and B

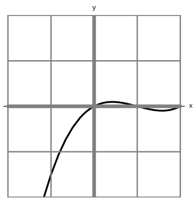
   (ii) Between which two points on the graph is the average rate of change the greatest?

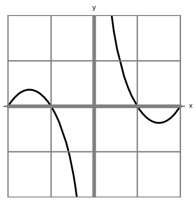
   (iii) Between which two points is the average rate of change most negative?

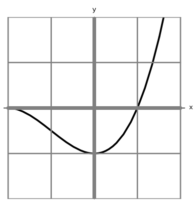
9. Each of the following functions is graphed somewhere below. Put the letter of the correct graph in the box next to the corresponding formula. Every graph is shown on the interval $-2 \le x \le 2$.

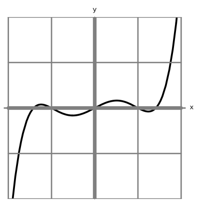
$\square$ $\frac{1}{4}\frac{1}{x(x+2)}$ $\square$ $\frac{1}{4}(x+2)^2(x-1)$ $\square$ $\frac{1}{4}\frac{(x^2-4)(x^2-1)}{x}$

A 

B 

C 

D 

E 

F