Publications and Preprints

with brief abstracts

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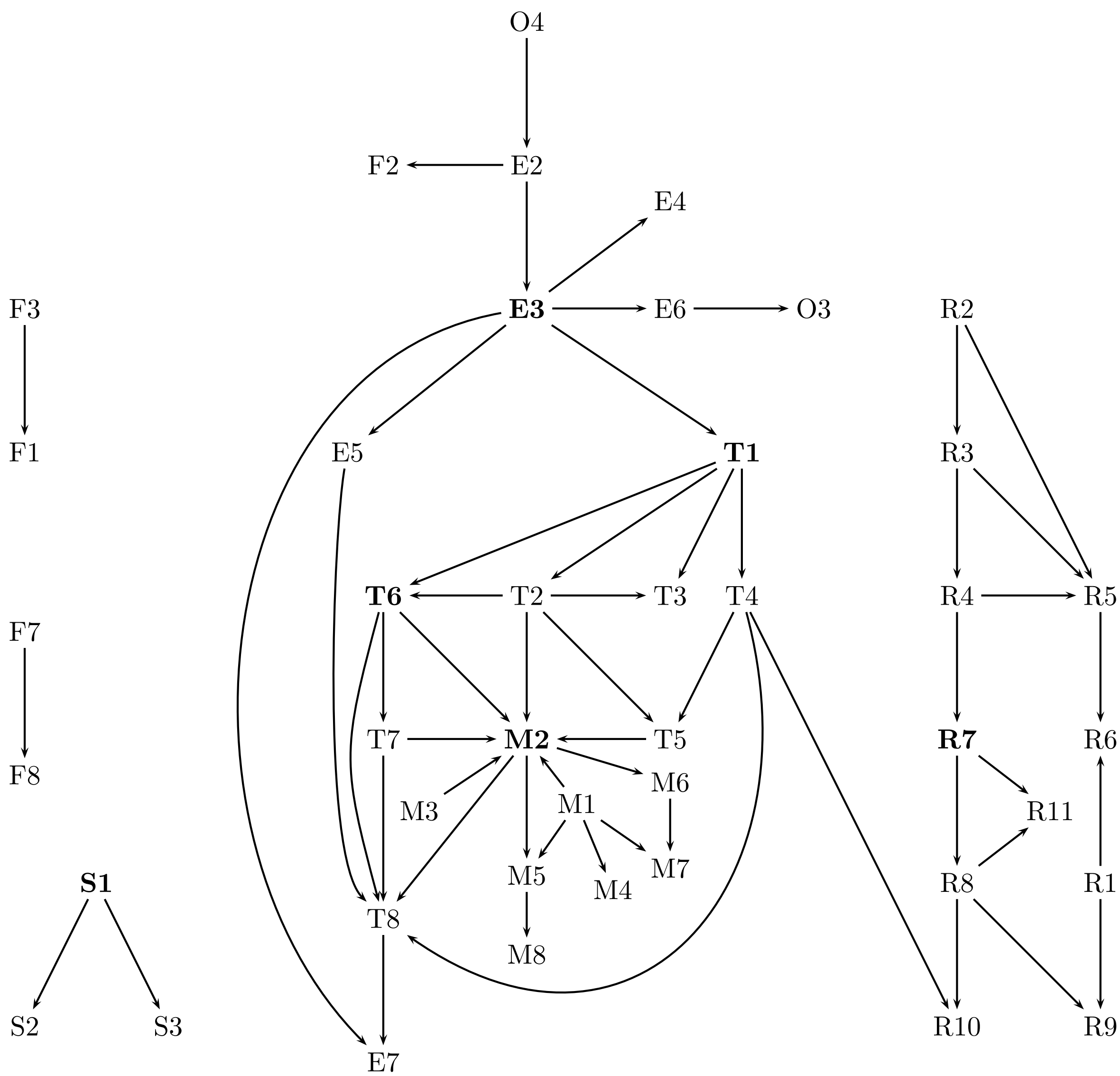
Primary Connections between Papers



E3. `Analysis of local obstructions` and `local excess intersection approach`

T1. `A sharp compactness theorem for genus-one pseudoholomorphic maps`

T6. `A smooth compactification of the Hilbert scheme of smooth genus-one curves in` $\mathbb{P}^n$

M2. `Proof of the BCOV prediction for the quintic threefold`

S1. `Normal crossings singularities for symplectic topology`

R7. `Real Gromov-Witten theory and enumerative geometry in all genera`

Real Gromov-Witten Theory and Enumerative Geometry

R1. M. Farajzadeh Tehrani and A. Zinger, *Counting Genus Zero Real Curves in Symplectic Manifolds*, Part II, math/1205.1809, 2 figures, Geom. Topol. 20 (2016), no. 2, 629-695

The second part explicitly describes, in the case of projective spaces, the orientations for the moduli spaces of real genus 0 maps introduced in the first part of the paper. Equivariant localization is then used to compare the real genus 0 invariants for the two standard involutions on the odd-dimensional projective spaces.

R2. P. Georgieva and A. Zinger, *The Moduli Space of Maps with Crosscaps: Fredholm Theory and Orientability*, math/1301.1074, 1 figure, Comm. Anal. Geom. 23 (2015), no. 3, 81–140

This paper proposes a fundamentally different approach around the two main obstacles to defining real GW-invariants, the orientability of the relevant moduli spaces and the existence of codimension-one boundary of these moduli spaces. Motivated by the string theory view of real Gromov-Witten invariants, we introduce moduli spaces of maps from surfaces with crosscaps, develop the relevant Fredholm theory, and resolve the orientability problem in this setting. This approach replaces symmetric surfaces as the domains of the maps by surfaces with crosscaps, which have much simpler codimension-one degenerations. As an immediate application, we describe a large collection of targets for which the genus 0 moduli spaces of real maps are orientable.

R3. P. Georgieva and A. Zinger, *The Moduli Space of Maps with Crosscaps: the Relative Signs of the Natural Automorphisms*, math/1308.1345, J. Symplectic Geom. 14 (2016), no. 2, 359–430

This paper determines the relative signs of the automorphisms of the moduli spaces of maps with crosscaps introduced in R2, without any global assumptions on the target manifold. As immediate applications, we describe sufficient conditions for the moduli spaces of real genus 1 maps and for real maps with separating fixed locus to be orientable. The sign computations in this paper also lead to an extension of recent Floer-theoretic applications of anti-symplectic involutions and to a related reformulation of these results in a more natural way.

R4. P. Georgieva and A. Zinger, *Orientability in Real Gromov-Witten Theory*, math/1308.1347, 25 pages, 3 figures

This paper describes a large collection of symplectic manifolds for which the moduli spaces of real maps of any genus are orientable, thus resolving the orientability problem for these targets. In contrast to the typical approaches to this problem, we do not compute the signs of any diffeomorphisms, but instead compare them. Many projective complete intersections, including the renowned quintic threefold, satisfy our topological conditions. We also study the orientability of the moduli spaces of real Hurwitz covers.

R5. P. Georgieva and A. Zinger, *Enumeration of Real Curves in $\mathbb{CP}^{2n-1}$ and a WDVV Relation for Real Gromov-Witten Invariants*, math/1309.4079, 7 figures, 3 tables, Duke Math. 166 (2017), no. 17, 3291-3347

This paper establishes a homology relation for the Deligne-Mumford moduli spaces of real curves, which lifts to a WDVV-type relation for real GW-invariants of real symplectic manifolds, and obtains a vanishing theorem for these invariants. For many real symplectic manifolds, these results reduce all genus 0 real invariants with conjugate pairs of constraints to genus 0 invariants with a single conjugate pair of constraints. For $\mathbb{CP}^{2n-1}$, they provide a complete recursion for counts of real rational curves with conjugate pairs of constraints and specify all cases when they are nonzero (thus providing nontrivial lower bounds in high-dimensional real algebraic geometry). In the case of $\mathbb{CP}^3$, these recursions imply that the real genus 0 invariants with conjugate point constraints are congruent to their complex analogues modulo 4.

R6. P. Georgieva and A. Zinger, *A Recursion for Counts of Real Curves in $C\mathbb{P}^{2n-1}$: Another Proof*, math/1401.1750, 3 figures, Internat. J. Math. 29 (2018), no. 4, 1850027, 21pp

This note provides another, more complex-geometric, proof of the recursion for real genus 0 GW-invariants with conjugate pairs of insertions in $\mathbb{CP}^{2n-1}$ which is a special case of the WDVV-type relation obtained in R5. The main part of this approach readily extends to real symplectic manifolds with empty real locus, but not to the general case.

R7. P. Georgieva and A. Zinger, *Real Gromov-Witten Theory in All Genera and Real Enumerative Geometry: Construction*, math/1504.06617, 47 pages, 5 figures, to appear in Ann. Math. 188 (2018), no. 3

This paper constructs positive-genus analogues of Welschinger's invariants for many real symplectic manifolds, including the odd-dimensional projective spaces and the renowned quintic threefold. In contrast to the previous attempts focused on the determinant lines of Fredholm operators, the approach to the orientability problem is based entirely on the topology of real bundle pairs over symmetric surfaces. We use it to endow the uncompactified moduli spaces of real maps from symmetric surfaces of all topological types with natural orientations and to verify that they extend across the codimension-one boundaries of these spaces, thus implementing a far-reaching proposal from C.-C. Liu's thesis.

R8. P. Georgieva and A. Zinger, *Real Gromov-Witten Theory in All Genera and Real Enumerative Geometry: Properties*, math/1507.06633, 56 pages, to appear in J. Symplectic Geom.

This paper studies the compatibility of the orientations on the moduli spaces of real maps constructed in R7 with the standard node-identifying immersion of Gromov-Witten theory. It also compares these orientations with alternative ways of orienting the moduli spaces of real maps that are available in special cases.

R9. P. Georgieva and A. Zinger, *Real Gromov-Witten Theory in All Genera and Real Enumerative Geometry: Computation*, math/1510.07568, 66 pages, 3 figures, to appear in J. Diff. Geom.

This paper applies the results of R8 to obtain quantitative and qualitative conclusions about the real Gromov-Witten invariants defined in R7. It describes large collections of real-orientable symplectic manifolds and shows that the real genus 1 Gromov-Witten invariants of sufficiently positive almost Kahler threefolds are signed counts of real genus 1 curves only and thus provide direct lower bounds for the counts of these curves in such targets. We specify real orientations on the real-orientable complete intersections in projective spaces and obtain equivariant localization data that computes the real invariants of projective spaces and determines the contributions from many torus fixed loci for other complete intersections.

R10. J. Niu and A. Zinger, *Lower Bounds for the Enumerative Geometry of Positive-Genus Real Curves*, math/1511.02206, 3 figures, 2 tables, Adv. Math. 339 (2018), no. 1, 191-247

This paper transforms the positive-genus real Gromov-Witten invariants of many real-orientable symplectic threefolds constructed in R7 into signed counts of curves. These integer invariants provide lower bounds for counts of real curves of a given genus that pass through conjugate pairs of constraints. We conclude with some implications for one- and two-partition Hodge integrals.

R11. P. Georgieva and A. Zinger, *On the Topology of Real Bundle Pairs over Nodal Symmetric Surfaces*, math/1512.07216, 3 figures, Topology Appl. 214 (2016), 109-126

This paper gives an alternative argument for the classification of real bundle pairs over smooth symmetric surfaces and extends this classification to nodal symmetric surfaces. It also classifies the homotopy classes of automorphisms of real bundle pairs over symmetric surfaces. The two main statements together describe the isomorphisms between real bundle pairs over symmetric surfaces up to deformation.

R12. P. Georgieva and A. Zinger, *Real Orientations, Real Gromov-Witten Theory, and Real Enumerative Geometry*, math/1512.07220, ERA MS 24 (2017), 87-99

This note is an overview of the construction of real Gromov-Witten theory in arbitrary genera in R7 and of its properties and applications obtained in R8-R10. It in particular highlights the principle of orienting the determinant of a differential operator relative to a suitable base operator and a real setting analogue of the (relative) spin structure of open Gromov-Witten theory.

R13. A. Zinger, *Real Ruan-Tian Perturbations*, math/1701.01420, 50 pages, 2 figures

This paper describes an analogue of Ruan-Tian deformations compatible with the construction of real Gromov-Witten invariants in arbitrary genera in R7 that enable a geometric definition of these invariants in many cases. The approach in this paper avoids the need for an embedding of the universal curve into a smooth manifold and systematizes the deformation-obstruction setup behind constructions of Gromov-Witten invariants.

General Symplectic Topology

S1. M. Farajzadeh Tehrani, M. McLean, and A. Zinger, *Normal Crossings Singularities for Symplectic Topology*, math/1410.0609v3, 65 pages, 4 figures, to appear in Adv. Math.

This paper introduces topological notions of symplectic normal crossing divisor and variety and show that they are equivalent, in a suitable sense, to the desired rigid notions. This partially answers Gromov's question on the feasibility of defining singular symplectic (sub)varieties and lays foundation for rich developments in the future.

S2. M. Farajzadeh Tehrani, M. McLean, and A. Zinger, *The Smoothability of Normal Crossings Singularities in Symplectic Topology*, math/1410.2573v2, 59 pages, 4 figures

We show that the direct topological analogue of the renown triple-point condition of algebraic geometry for the smoothability of normal crossings varieties is the only obstruction in the category of symplectic normal crossings varieties introduced in S1. Every smooth fiber of the families of smoothings we describe provides a multifold analogue of the now classical (two-fold) symplectic sum construction; we thus establish an old suggestion of Gromov in a strong form.

S3. M. Farajzadeh Tehrani and A. Zinger, *Normal Crossings Degenerations of Symplectic Manifolds*, math/1603.07661, 65 pages, 2 figures

We use local Hamiltonian torus actions to degenerate a symplectic manifold to a normal crossings symplectic variety in a smooth one-parameter family. This construction, motivated in part by the Gross-Siebert and B. Parker's programs, contains a multifold version of the usual (two-fold) symplectic cut construction and in particular splits a symplectic manifold into several symplectic manifolds containing normal crossings symplectic divisors with shared irreducible components in one step.

S4. M. Farajzadeh Tehrani, M. McLean, and A. Zinger, *Singularities and Semistable Degenerations for Symplectic Topology*, math/1707.01464, 2 figures, C. R. Math. Acad. Sci. Paris 356 (2018), no. 4, 420-432

This note overviews the introduction of a symplectic notion of normal crossings singularity in S1, the topological characterization of smoothable singularities in S2, and the multifold symplectic degeneration construction in S3. It in particular highlights the principle of viewing normal crossings symplectic varieties as deformation equivalence classes, instead of individual spaces.

Foundations of Gromov-Witten Theory

F1. A. Zinger, *Pseudocycles and Integral Homology*, math/0605535, 2 figures, Trans. AMS 360 (2008), no. 5, 2741-2765

This paper describes a natural isomorphism between the set of equivalence classes of pseudocycles and the integral homology groups of a smooth compact manifold. This isomorphism has been used widely in symplectic topology. Along the way, observations are made regarding the topology of neighborhoods of images of smooth maps and the singular homology of smooth manifolds (in particular, establishing the existence of what Simons-Sullivan later call k-good neighborhoods in more restricted settings).

F2. A. Zinger, *A Comparison Theorem for Gromov-Witten Invariants in the Symplectic Category*, math/0807.0805, Adv. Math. 228 (2011), no. 1, 535–574

This paper compares GW-invariants of a symplectic manifold and a symplectic submanifold whenever all constrained stable maps to the former are contained in the latter to first order. Various special cases of the main comparison theorem in this paper have long been used in the algebraic category; some of them have also appeared in the symplectic setting. Combined with the inherent flexibility of the symplectic category, the main theorem leads to a confirmation of Pandharipande's Gopakumar-Vafa prediction for GW-invariants of Fano classes in 6-dimensional symplectic manifolds. The proof of the main theorem uses deformations of the Cauchy-Riemann equation that respect the submanifold and Carleman Similarity Principle for solutions of perturbed Cauchy-Riemann equations.

F3. A. Zinger, *On Transverse Triangulations*, math/1012.3979, Münster J. Math. 5 (2012), 99–106

This note shows that every smooth manifold admits a smooth triangulation transverse to a given smooth map. As pointed out by M. Kreck, this fact, assumed in F1 to be well-known, had been established by Scharlemann only under the assumption that the map is proper. This note provides a completely different, more direct, argument that removes the need for the properness assumption.

F4. A. Zinger, *The Determinant Line Bundle for Fredholm Operators: Construction, Properties, and Classification*, math/1304.6368, 52 pages, Math. Scand. 118 (2016), no. 2, 203–268

This paper provides a detailed construction of a system of compatible determinant line bundles over spaces of Fredholm operators, fully verifies that this system satisfies a number of important properties, and includes explicit formulas for all relevant isomorphisms between these line bundles. It also completely describes all possible systems of compatible determinant line bundles and compares the conventions and approaches used elsewhere in the literature.

F5. M. Farajzadeh Tehrani and A. Zinger, *On Symplectic Sum Formulas in Gromov-Witten Theory*, math/1404.1898, 91 pages, 6 figures

This manuscript provides an extensive overview of the setting of the symplectic sum formula for GW-invariants. We also analyze and compare the two analytic approaches aimed at tackling the issues needed to be resolved in establishing it.

F6. M. Farajzadeh Tehrani and A. Zinger, *Absolute vs. Relative Gromov-Witten Invariants*, math/1405.2045, 11 figures, 4 tables, J. Symplectic Geom. 14 (2016), no. 4, 1189–1250

This paper compares the absolute and relative GW-invariants of compact symplectic manifolds when they might be expected to be equal. We show that these invariants are indeed equal, except in a narrow range of dimensions of the target and genera of the domains, and provide examples when they fail to be the same.

F7. M. Farajzadeh Tehrani and A. Zinger, *On the Rim Tori Refinement of Relative Gromov-Witten Invariants*, math/1412.8204, 43 pages

This paper constructs Ionel-Parker's proposed refinement of the standard relative Gromov-Witten invariants in terms of abelian covers of the symplectic divisor and discusses in what sense it gives rise to invariants. We use it to obtain some vanishing results for the standard relative Gromov-Witten invariants.

F8. M. Farajzadeh Tehrani and A. Zinger, *On the Refined Symplectic Sum Formula for Gromov-Witten Invariants*, math/1412.8205, 54 pages

We describe the extent to which Ionel-Parker's proposed refinement of the standard relative Gromov-Witten invariants sharpens the usual symplectic sum formula. The key product operation on the target spaces for the refined invariants is specified in terms of abelian covers of symplectic divisors, making it suitable for studying from a topological perspective. We give several qualitative applications of this refinement, which include vanishing results for Gromov-Witten invariants.

F9. A. Zinger, *Notes on J-Holomorphic Maps*, math/1706.00331, 52 pages, 11 figures

These notes present a systematic treatment of local properties of J-holomorphic maps and of Gromov's convergence for sequences of such maps, specifying the assumptions needed for all statements. In particular, only one auxiliary statement depends on the manifold being symplectic.

F10. A. Zinger, *The Virtual Fundamental Class in Gromov-Witten Theory: the Li-Tian Construction and Beyond*, mostly expository manuscript in preparation

The purposes of this manuscript are to provide an extensive exposition of the Li-Tian construction of the virtual fundamental class in GW-theory and to describe its applications. At the present, this manuscript is in several pieces addressing different aspects of the Li-Tian construction and totaling over 100 pages.

Mirror Symmetry

M1. A. Zinger, *Genus-Zero Two-Point Hyperplane Integrals in the Gromov-Witten Theory*, math/0705.2397, 4 figures, Comm. Analysis Geom. 17 (2010), no. 5, 1–45

This paper explicitly computes two-point analogues of the one-point integrals that are central to the proof of the genus 0 mirror symmetry for a quintic threefold. These integrals are in a sense of arithmetic genus 1 and are an essential ingredient in the genus 1 computation in M2.

M2. A. Zinger, *The Reduced Genus-One Gromov-Witten Invariants of Calabi-Yau Hypersurfaces*, math/0705.2397, 5 figures, JAMS 22 (2009), no. 3, 691–737

The reduced genus 1 GW-invariants of Calabi-Yau projective hypersurfaces are computed explicitly by using T5 to reduce the computation to $\overline{\mathfrak{M}}^0_{1,k}(\mathbb{P}^n,d)$ and T6 to apply the classical localization theorem. The 1993 Bershadsky-Cecotti-Ooguri-Vafa (BCOV) prediction for the standard genus 1 GW-invariants of the quintic threefold is confirmed as a special case. The summation over the fixed loci is handled by relating the problem to previously carried out genus 0 localization computations and then extracting the non-equivariant part.

M3. D. Zagier and A. Zinger, *Some Properties of Hypergeometric Series Associated with Mirror Symmetry*, math/0710.0889, *Modular Forms and String Duality*, Fields Inst. Commun. 54 (2008), 163–177

This paper shows that certain hypergeometric series playing a prominent role in mirror symmetry for Calabi-Yau hypersurfaces possess a variety of interesting properties. While these properties appear intriguing in their own right, some of them are also used in M2 to compute the reduced genus 1 Gromov-Witten invariants of Calabi-Yau projective hypersurfaces and to verify the 1993 BCOV prediction for the quintic threefold.

M4. A. Popa and A. Zinger, *Mirror Formulas for Closed, Open, and Unoriented Gromov-Witten Invariants*, math/1010.1946, 2 figures, 5 tables, Adv. Math. 259 (2014), 448–510

The first part of this paper extends the two-point mirror formulas of M1 to projective complete intersections, describing the relevant structure coefficients in a far greater detail, as needed for the applications in the second and third parts. The latter confirm Walcher's mirror symmetry conjectures for the annulus and Klein bottle GW-invariants of Calabi-Yau complete intersection threefolds, using different techniques to handle the graph sums that appear in the two cases.

M5. A. Zinger, *The Genus 0 Gromov-Witten Invariants of Projective Complete Intersections*, math/1106.1633, 3 figures, 4 tables, Geom. Top. 18 (2014), no. 2, 1035-1114

This paper describes the structure of mirror formulas for the genus 0 GW-invariants of projective complete intersections with any number of marked points and provides an explicit algorithm for obtaining the relevant structure coefficients. The main theorem leads to bounds on the growth of these invariants, generalizing those predicted by R. Pandharipande, and to completely unexpected vanishing results for GW-invariants.

M6. Y. Cooper and A. Zinger, *Mirror Symmetry for Stable Quotients Invariants*, math/1201.6350, 6 figures, Mich. Math. J. 63 (2014), no. 3, 571–621

This paper establishes a mirror formula for the stable quotient invariants of the quintic threefold defined by Marian-Oprea-Pandharipande and shows that they are in fact closer to the complex geometry of the mirror quintic than the standard Gromov-Witten invariants. The argument is an intricate twist on Givental's proof of mirror symmetry for GW-invariants and involves bootstrapping from the Fano to the Calabi-Yau cases and flipping the use of the two variations of Givental's approach in the Fano and Calabi-Yau cases. As a by-product, the argument produces mirror formulas for certain twisted stable quotients Hurwitz numbers.

M7. A. Zinger, *Double and Triple Givental's J-Function for Stable Quotients Invariants*, math/1305.2142, 9 figures, Pacific J. Math. 272 (2014), no. 2, 439–507

This paper runs the one-point formulas for the stable quotient invariants obtained in M6 through the principle of M1 to explicitly compute two- and three-point stable quotient invariants of projective complete intersections and some twisted Hurwitz numbers. This computation in particular implies that these invariants satisfy the string relation in the Fano cases, but not in the Calabi-Yau cases.

M8. A. Zinger, *Energy bounds and vanishing results for the Gromov-Witten invariants of the projective space*, math/1801.07242, 57 pages, 4 figures

This paper builds on M5 to describe generating functions for arbitrary-genus Gromov-Witten invariants of the projective space with any number of marked points explicitly. The structural portion of this description gives rise to uniform energy bounds and vanishing results for these invariants. They suggest deep conjectures relating Gromov-Witten invariants of symplectic manifolds to the energy of pseudo-holomorphic maps and the expected dimension of their moduli space.

M9. A. Zinger, *Equivariant Localization and Mirror Symmetry*, expository manuscript in preparation

This manuscript, currently at 80 pages, started as a streamlined version of R. Pandharipande's expository notes on Givental's proof of mirror symmetry for the genus 0 Gromov-Witten invariants of projective hypersurfaces. It now includes a thorough introduction to equivariant cohomology and a proof of the Atiyah-Bott Equivariant Localization Theorem.

Topology of Moduli Spaces of Pseudoholomorphic Maps

T1. A. Zinger, *A Sharp Compactness Theorem for Genus-One Pseudo-Holomorphic Maps*, math/0406103, 7 figures, Geom. Top. 13 (2009), no. 5, 2427–2522

This paper gives a sharp version of the stable-map compactness theorem in the genus 1 case. If (X, ω, J) is a symplectic manifold satisfying certain regularity conditions, the described "main component" of the moduli space of genus 1 stable maps is the closure of the space of maps with smooth domains and carries a rational fundamental class. The compactification theorem is proved by adopting the gluing procedure of E2 and using the power series of E3.

T2. A. Zinger, *On the Structure of Certain Natural Cones over Moduli Spaces of Genus-One Holomorphic Maps*, math/0406104, 3 figures, Adv. Math. 214 (2007), no. 2, 878–933

It is shown in this paper that certain naturally arising cones over the "main component" of a moduli space of genus 1 stable maps have a well-defined euler class, provided the symplectic manifold (X, ω, J) is sufficiently regular. This result is obtained by adapting the power series of E3 for the behavior of derivatives of holomorphic maps under gluing to holomorphic bundle sections.

T3. J. Li and A. Zinger, *On Gromov-Witten Invariants of a Quintic Threefold and a Rigidity Conjecture*, math/0406105, 5 figures, Pacific J. Math 233 (2007), no. 2, 417–480

This paper rederives a relation for genus 1 GW-invariants obtained in T5 in the special case of a quintic threefold. The derivation is conditioned on a rigidity conjecture for curves in a quintic threefold and in a sense provides additional evidence for the conjecture. This paper's approach is more direct and geometric than that in T5. It relies heavily on T2 and T1, but not on T4 or the generalization of T2 that constitutes most of T5.

T4. A. Zinger, *Reduced Genus-One Gromov-Witten Invariants*, math/0507103, 5 figures, J. Diff. Geom. 83 (2009), no. 2, 407–460

In this paper, some of the constructions of T1 are generalized to moduli spaces of perturbed, in a restricted way, J-holomorphic maps. This generalization implies that the main component of the moduli space of J-holomorphic maps carries a virtual fundamental class and defines symplectic invariants. These truly genus 1 invariants differ from the standard genus 1 GW-invariants by a combination of the genus 0 invariants.

T5. J. Li and A. Zinger, *On the Genus-One Gromov-Witten Invariants of Complete Intersections*, math/0507104, 4 figures, 1 table, J. Diff. Geom. 82 (2009), no. 3, 641–690

Most of this paper is devoted to extending the construction of T2 to moduli spaces of perturbed, in a restricted way, J-holomorphic maps. An easy consequence of this extension is a relation between the reduced genus 1 GW-invariants of complete intersections and the reduced genus 1 GW-invariants of the ambient projective space which parallels a well-known relation for the genus 0 invariants. As an application, the standard genus 1 GW-invariants of complete intersections can be expressed in terms of the genus 0 and genus 1 GW-invariants of projective spaces, solving the genus 1 case of a fundamental problem in GW-theory. A relationship for higher-genus invariants is conjectured as well.

T6. R. Vakil and A. Zinger, *A Desingularization of the Main Component of the Moduli Space of Genus-One Stable Maps into* $\mathbb{P}^n$, math/0603353, 13 figures, Geom. Top. 12 (2008), no. 1, 1–95

Making use of structural descriptions in T2, T1, and E5, this paper constructs a desingularization of the "main component" $\overline{\mathfrak{M}}^0_{1,k}(\mathbb{P}^n,d)$ of the moduli space of genus 1 stable maps into the complex projective space $\mathbb{P}^n$ and a desingularization of a natural cone over $\overline{\mathfrak{M}}^0_{1,k}(\mathbb{P}^n,d)$. Such a desingularization is useful for integrating natural cohomology classes on $\overline{\mathfrak{M}}^0_{1,k}(\mathbb{P}^n,d)$ via equivariant localization and thus has applications for computing genus 1 Gromov-Witten invariants of complete intersections and genus 1 enumerative invariants of projective spaces. As a bonus, we obtain desingularizations of natural sheaves over moduli spaces of genus 1 stable maps.

T6b. Overview (independent), 2 figures, ERA AMS, 13 (2007), 53-59

This short announcement describes the results of the paper from the more classical algebraic geometry perspective of a smooth compactification for the Hilbert scheme of smooth genus 1 curves in $\mathbb{P}^n$. Parallels are drawn with the space of complete plane conics, and the highlights of the desingularization construction are illustrated on the space of smooth plane cubics.

T6c. Extended Abstract, Oberwolfach Report 27/2006, 1643-1645

This extended abstract for Vakil's Oberwolfach talk describes the steps in the desingularization construction and some of the properties of the desingularized moduli space.

T7. A. Zinger, *Intersections of Tautological Classes on Blowups of Moduli Spaces of Genus-One Curves*, math/0603357, 4 figures, Mich. Math. 55 (2007), no 3, 535-560

Two recursions for computing top intersections of tautological classes on blowups of moduli spaces of genus 1 curves are obtained in this paper. One of these recursions is analogous to the well-known string equation. As shown in T6 and T5, these numbers are useful for computing genus 1 enumerative invariants of projective spaces and Gromov-Witten invariants of complete intersections.

T8. A. Zinger, *Standard vs. Reduced Genus-One Gromov-Witten Invariants*, math/0706.0715, 4 figures, Geom. Top. 12 (2008), no. 2, 1203–1241

The difference between the standard genus 1 GW-invariants and the reduced genus 1 GW-invariants defined in T4 is expressed in terms of genus 0 GW-invariants. The coefficients involve blowups of moduli spaces of genus 1 curves constructed in T6 and are computable through the recursive formulas obtained in T7. Along with T6 and T5, this paper makes it possible to compute the standard genus 1 GW-invariants of complete intersections. As an application, an explicit closed formula for the standard genus 1 GW-invariants of a Calabi-Yau projective hypersurface (of any dimension) is deduced from a closed formula for the reduced invariants obtained in M2. A recent mirror symmetry prediction for a sextic fourfold is thus verified as a special case; the higher-dimensional cases go beyond any predictions.

Local Excess Intersection Approach

E1. A. Zinger, *Completion of Katz-Qin-Ruan's Enumeration of Genus-Two Plane Curves*, math/0201216, 3 figures, 1 table, J. Algebraic Geom. 13 (2004), no. 3, 547–561

In this paper, a formula for the number of genus 2 degree d plane curves that pass through $3d-2$ general points and have a fixed complex structure on the normalization is obtained. This is achieved by completing Katz-Qin-Ruan's work on the same question. The resulting formula agrees with the corresponding formula in E3, which is obtained in a completely different way.

E2. A. Zinger, *Enumerative vs. Symplectic Invariants and Obstruction Bundles*, math/0201255, J. Symplectic Geom. 2 (2004), no. 4, 445–543

This paper presents in detail a gluing construction for pseudoholomorphic maps in symplectic topology, especially in the presence of an obstruction bundle. The explicit nature of this construction leads to two types of power series in E3, which are also used in an essential way in several other papers. The main motivation is to try to compare the symplectic and enumerative invariants of algebraic manifolds.

E3. A. Zinger, *Enumeration of Genus-Two Curves with a Fixed Complex Structure in $\mathbb{P}^2$ and $\mathbb{P}^3$*, math/0201254, 3 tables, 2 figures, J. Diff. Geom. 65 (2003), no. 3, 341–467

This paper uses the explicit gluing construction of E2 to obtain power series that describe the behavior of all derivatives of rational holomorphic maps into $\mathbb{P}^n$ under gluing and obstructions to gluing positive-genus maps in some situations. It also introduces the notion of contribution of a (possibly open) stratum of the zero set of a vector bundle section to the euler class of the bundle and relates this contribution to the number of zeros of an affine bundle map over the closure of the stratum. The main applications are formulas for the number of genus 2 curves with a fixed complex structure in $\mathbb{P}^2$ and $\mathbb{P}^3$. Other applications include formulas for the number of rational curves with a cusp in $\mathbb{P}^2$ and $\mathbb{P}^3$.

E4. A. Zinger, *Enumeration of Genus-Three Plane Curves with a Fixed Complex Structure*, math/0203058, 3 tables, J. Algebraic Geom. 14 (2005), no. 1, 35–81

Using the approach of E3, this paper determines the number of genus 3 degree d plane curves that pass through general $3d$–4 points and have a fixed non-hyperelliptic complex structure on the normalization. An intermediate result is a formula for the number of rational plane curves with a $(3,4)$-cusp.

E5. A. Zinger, *Enumeration of One-Nodal Rational Curves in Projective Spaces*, math/0204236, 9 figures, 1 table, Topology 43 (2004), no. 4, 793–829

This paper gives a formula computing the number of genus 1 curves with any fixed j-invariant that pass through an appropriate collection of constraints in a complex projective space. Two of the key ingredients are the topological method and the power series for derivatives of E3.

E6. A. Zinger, *Counting Rational Curves of Arbitrary Shape in Projective Spaces*, math/0210146, 15 figures, 5 tables, Geom. Top. 9 (2005), 571–697

This paper describes an algorithm for solving a large class of enumerative problems concerning rational curves in projective spaces by significantly extending the applicability of the topological method and of the power series for derivatives of E3. Its application in each specific case requires only general understanding of the topology of moduli space of holomorphic maps. This paper's approach is demonstrated by enumerating one-component rational curves in $\mathbb{P}^3$ that have a triple point, that have a tacnodal point, and cuspidal curves in $\mathbb{P}^n$.

E7. R. Pandharipande and A. Zinger, *Enumerative Geometry of Calabi-Yau 5-Folds*, math/0802.1640, 6 figures, 3 tables, *New Developments in Algebraic Geometry, Integrable Systems and Mirror Symmetry*, ASPM 59 (2010), 239-288; Appendix, 6 pages

Enumerative geometry for Calabi-Yau 5-folds is defined from Gromov-Witten invariants, following up on Gopakumar-Vafa and Klemm-Pandharipande predictions for CY 3-folds and 4-folds, respectively. A degree scaling for contributions of multiple covers of rational curves to genus 1 GW-invariants is established via analysis of local obstructions motivated by T1, E2, and E3. Integrality tests are carried out for a septic hypersurface in $\mathbb{P}^6$, using closed formulas from T8 and M1, and for the total space of the bundle $3\mathcal{O}(-1)$ over $\mathbb{P}^2$; in the latter case, G. Martin has conjectured an explicit formula for all genus 1 counts.

Miscellaneous

O1. M. Kalka, E. Mann, D. Yang, and A. Zinger, *The Exponential Decay Rate of the Lower Bound for the First Eigenvalue of Compact Manifolds*, 1 figure, Internat. J. Math. 8 (1997), no. 3, 345–355

This paper provides the optimal exponential decay rate of the lower bound for the first positive eigenvalue of the Laplacian operator on a compact Riemannian manifold with a negative lower bound on the Ricci curvature and with large diameter.

O2. A. Zinger, *Enumerative Algebraic Geometry via Techniques of Symplectic Topology and Analysis of Local Obstructions*, PhD thesis, MIT 2002, 240 pages, 8 tables

This dissertation contains foundations of the Local Excess Intersection Approach and its applications in Enumerative Geometry, including in finite-dimensional singular and infinite-dimensional Fredholm settings. It incorporates the papers E1-E4 and includes a detailed introduction.

O3. A. Zinger, *Counting Plane Rational Curves: Old and New Approaches*, math/0507105, 32 pages, 2 figures, 2 tables

These notes are intended as an easy-to-read introduction to a classical and a modern approach in enumerative algebraic geometry. In particular, the numbers n_3 and n_4 of plane rational cubics through 8 points and of plane rational quartics through 11 points are determined via the classical approach of counting curves. The computation of the latter number also illustrates the topological method of E3. The arguments used in the computation of the number n_4 extend easily to counting plane curves with 2 or 3 nodes, for example. Finally, an inductive formula for the number n_d of plane degree d rational curves passing through $3d-1$ points is derived via the modern approach of counting stable maps.

O4. A. Zinger, *Basic Riemannian geometry and Sobolev estimates used in symplectic topology*, math/1012.3980, 33 pages, 3 figures

This note collects a number of standard statements in Riemannian geometry and in Sobolev-space theory that play a prominent role in analytic approaches to symplectic topology. These include relations between connections and complex structures, estimates on exponential-like maps, and dependence of constants in Sobolev and elliptic estimates.

O5. J. Chen and A. Zinger, *On the Robustness of Extortionate Strategies in Iterated Prisoner's Dilemma*, 4 figures, J. Theoret. Biol. 357 (2014), 46–54

This paper mathematically confirms the predicted robustness of the strategies for Iterated Prisoner's Dilemma discovered by Press-Dyson by reducing the issue at hand to a small number of simple checks.

O6. A. Zinger, *Some Conjectures on the Asymptotic Behavior of Gromov-Witten Invariants*, math/1610.02971v2, 26 pages, 1 figure

This note shares some observations and speculations concerning the asymptotic behavior of Gromov-Witten invariants. They may be indicative of some deep phenomena in symplectic topology that in full generality are outside of the reach of current techniques. On the other hand, many interesting cases can perhaps be treated via combinatorial techniques.

O7. A. Zinger, *Some questions in the theory of pseudoholomorphic curves*, math/1805.09647, 28 pages, 4 figures

This survey article, in honor of G. Tian's 60th birthday, is inspired by R. Pandharipande's 2002 note highlighting research directions central to Gromov-Witten theory in algebraic geometry and by G. Tian's complex-geometric perspective on pseudoholomorphic curves that lies behind many important developments in symplectic topology since the early 1990s.

O8. A. Zinger, *Foundations of Smooth Manifolds and Vector Bundles*, expository manuscript in preparation

This manuscript, currently at 108 pages, started as a supplement to Warner's differentiable geometry textbook for the core graduate geometry course at Stony Brook. It is in the same laconic style as Warner's book, but has a more modern focus (in particular includes vector bundles) and contains many more examples.