Lecture #14
MAT 123

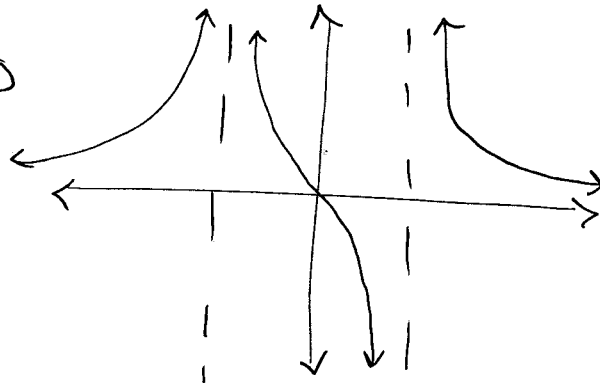## Rational Functions

Rational Function   "a function over a function"

$$y = \frac{f(x)}{g(x)} \quad , \quad g(x) \neq 0$$

$$y = \frac{x-3}{\boxed{x+2}} \longrightarrow \text{if } x = -2$$

$$x + 2 = 0$$

so when $x = -2$ y is undefined
also there is a vertical asymptote
when $x = -2$

$$y = \frac{3x-5}{(x+1)(x-1)}$$

* To find vertical asymptotes you set the denominator $= 0$ and solve.

Vertical asymptotes
at : $x + 1 = 0 \Rightarrow x = -1$
       $x - 1 = 0 \Rightarrow x = 1$

horizontal asymptote → look at the end behavior.
a.k.a. we need to look at the terms of highest degree.

$$y = \frac{ax^m + \ldots}{bx^n + \ldots}$$

** 3 options: ① $m > n \Rightarrow$ No horizontal asymptotes
              ② $n > m \Rightarrow$ H.A. at $y = 0$ ("x-axis")
              ③ $m = n \Rightarrow$ H.A. at $y = \frac{a}{b}$, a,b are the coefficients

example:

$$y = \frac{3x^4 + 2x^2 + 1}{5x^3 - 6x + 10}$$

Find horizontal asymptote:
   look at the terms of highest degree only

$$\frac{3x^{④}}{5x^{③}} \quad \begin{array}{c} 4 > 3 \\ m > n \end{array} \quad \text{so NO horizontal asymptote.}$$

example: $y = \dfrac{(3x^4) + 2x^3 + 1}{(5x^4) - 6x + 10}$

$$\frac{3x^{④}}{5x^{④}} \quad \begin{array}{c} 4 = 4 \\ m = n \end{array} \quad \text{So H.A. @ } y = \frac{3}{5}$$

example: $y = \dfrac{(3x^4) + 2x^3 + 1}{(5x^5) - 6x + 10}$

$$\frac{3x^{④}}{5x^{⑤}} \quad \begin{array}{c} 4 < 5 \\ m < n \end{array} \quad \text{so H.A. @ } y = 0 \\ (x\text{-axis})$$

ex: $y = \dfrac{x - 2}{(x+3)(x-4)}$ \qquad find V.A and H.A.

V.A. $(x+3)(x-4) = 0$

$$\begin{array}{c|c} x+3 = 0 & x - 4 = 0 \\ x = -3 & x = 4 \end{array}$$

$$\boxed{\text{V.A.} \begin{array}{c} x = -3 \\ x = 4 \end{array}}$$

H.A. $y = \dfrac{x - 2}{(x+3)(x-4)}$

$y = \dfrac{(x) - 2}{(x^2) - x - 12}$

$\dfrac{x^{①}}{x^{②}} \quad \begin{array}{c} 1 < 2 \\ m < n \end{array}$

H.A. @ $\boxed{y = 0}$

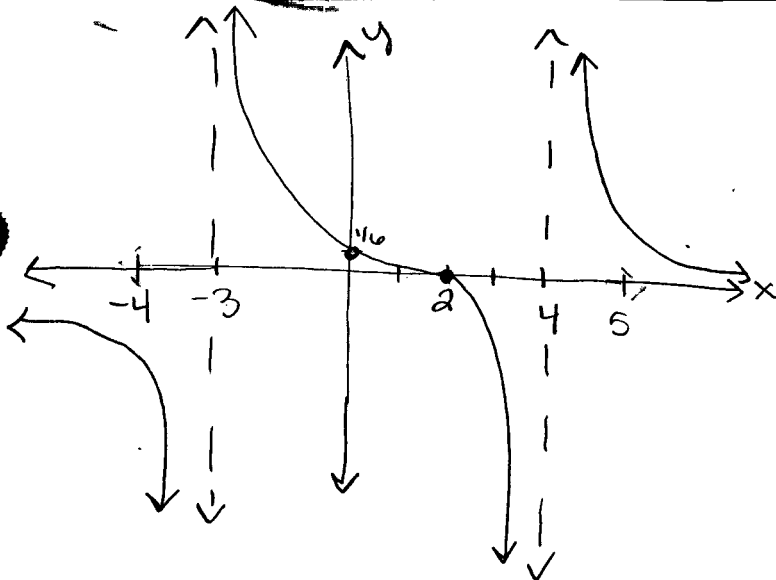x-intercept at $x = 2$ $\qquad$ numerator = 0

$x - 2 = 0$
$x = 2$

y-intercept. $\qquad$ plug in for x

$y = \dfrac{0 - 2}{(0+3)(0-4)} = \dfrac{-2}{-12} = \dfrac{1}{6}$

Sketch using our information!

test values into the function to help you figure out the shape.

Graph of $y = \dfrac{x-2}{(x+3)(x-4)}$

Practice: $y = \dfrac{x-4}{x+2}$

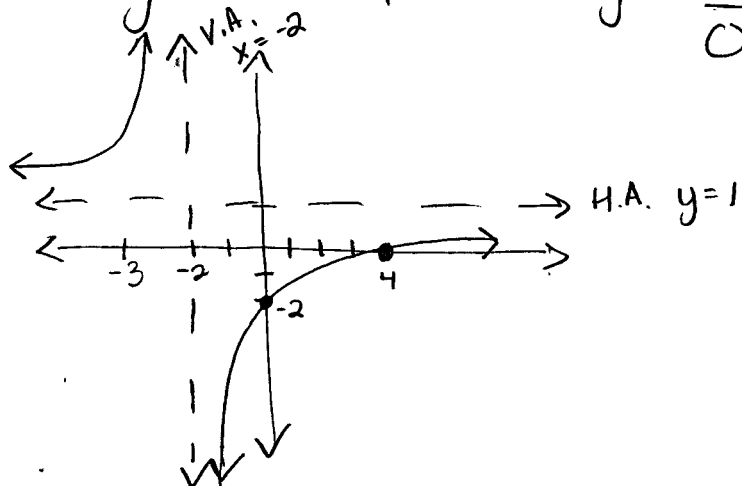H.A.: $\dfrac{x^1}{x^1}$ $1=1$ $m=n$ H.A. at $y = \dfrac{1}{1} = 1$

$\boxed{y=1}$

V.A: $x + 2 = 0$

$\boxed{x = -2}$

x-intercept: $x - 4 = 0$

$\boxed{x = 4}$

y-intercept: $y = \dfrac{0-4}{0+2} = \dfrac{-4}{2} = -2$

$\boxed{y = -2}$

H.A. $y=1$

test: $x = -3$

$y = \dfrac{-3-4}{-3+2} = \dfrac{-7}{-1} = 7$

practice: $y = \dfrac{3x-5}{x+1}$

V.A. $x+1=0$
$\boxed{x=-1}$

H.A. $\dfrac{3x^{\cancel{1}}}{x^{\cancel{1}}}$ $1=1$

H.A. @ $y = \dfrac{3}{1} = 3$
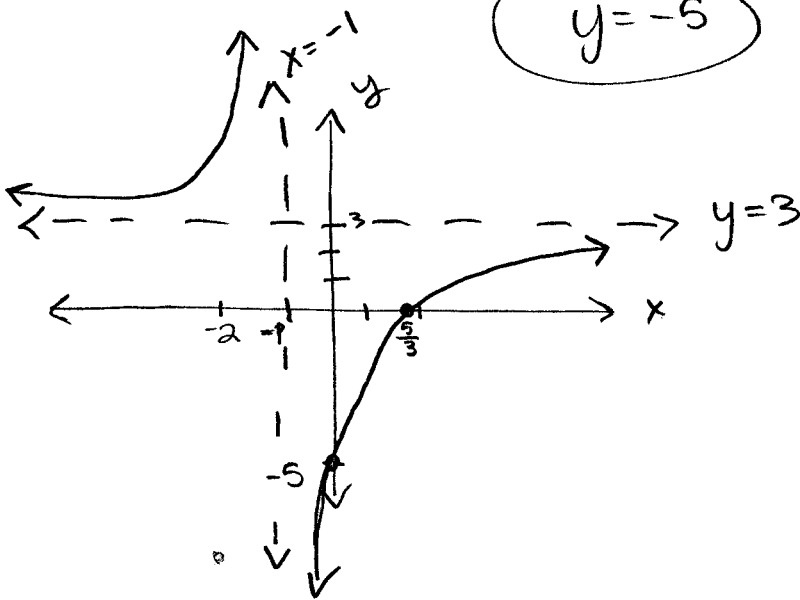
$\boxed{y=3}$

x-intercept: $3x-5=0$
$3x=5$
$\boxed{x=\dfrac{5}{3}} = 1.\overline{6}$

y-intercept: $y = \dfrac{3(0)-5)}{0+1} = \dfrac{-5}{1} = -5$

$\boxed{y=-5}$

test: $x=-2$
$y = \dfrac{3(-2)-5}{-2+1} = \dfrac{-11}{-1} = 11$
↑
positive

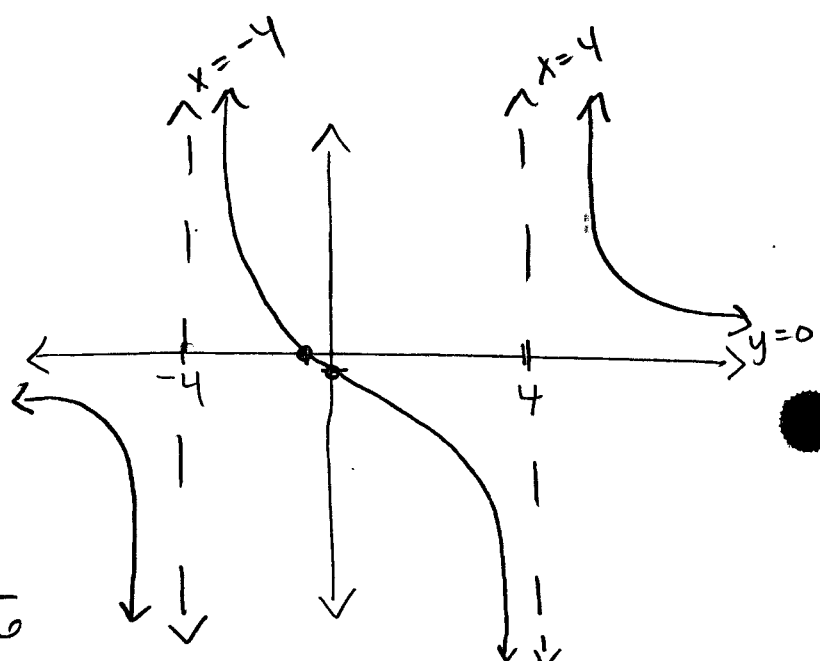practice: $y = \dfrac{2x+1}{(x-4)(x+4)}$

V.A. $(x-4)(x+4)=0$
$x=4, x=-4$

H.A. $\dfrac{2x'}{x^2}$ $\begin{array}{c}1<2\\ m<n\end{array}$
H.A. @ $y=0$

x-int: $2x+1=0$
$x=-1/2$

y-int: $y = \dfrac{2(0)+1}{(-4)(4)} = \dfrac{1}{-16}$ $y = \dfrac{-1}{16}$
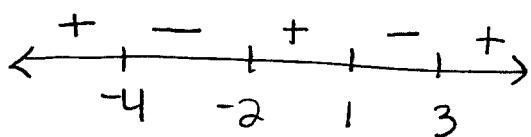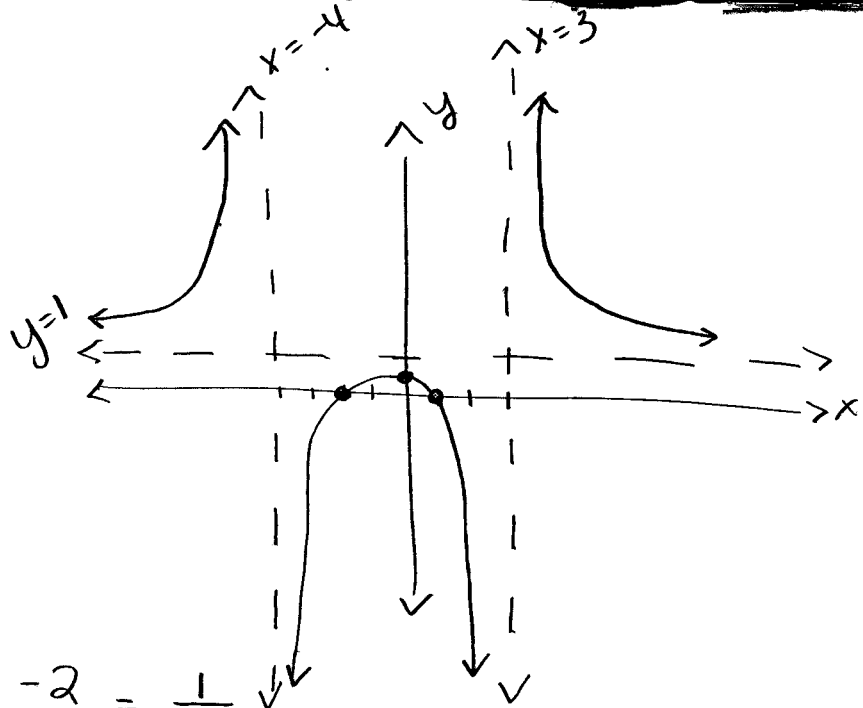
practice: $y = \dfrac{(x-1)(x+2)}{(x+4)(x-3)}$

- V.A.: $(x+4)(x-3) = 0$

  $x = -4$

  $x = 3$

  H.A.: $\dfrac{x^2}{x^2}$ $y = 1$

  x-int: $(x-1)(x+2) = 0$

  $x = 1$

  $x = -2$

  y-int: $y = \dfrac{(-1)(2)}{(4)(-3)} = \dfrac{-2}{-12} = \dfrac{1}{6}$

  $y = \frac{1}{6}$

$\xleftarrow{\hspace{1cm}} \; + \; \underset{-4}{|} \; - \; \underset{-2}{|} \; + \; \underset{1}{|} \; - \; \underset{3}{|} \; + \; \xrightarrow{\hspace{1cm}}$

✱ test values to help you find the shape!

Summary: To find:

① Horizontal Asymptote: look at terms of highest degree:

3 cases

$y = \dfrac{ax^m + \dots}{bx^n + \dots}$

① $m > n$: none

② $m < n$: $y = 0$

③ $m = n$: $y = a/b$

② Vertical Asymptotes: set denominator $= 0$ and solve.

③ X-intercepts: set numerator $= 0$ and solve

④ y-intercepts: plug in 0 for x, then solve for y.